SACLAY-T09/158
SISSA–64/2009/EP

Flavour violation in supersymmetric SO(10) unification with a type II seesaw mechanism

Lorenzo Calibbi[a], Michele Frigerio[b], Stéphane Lavignac[b] and Andrea Romanino[a]

[a] *SISSA/ISAS and INFN, I–34013 Trieste, Italy*
[b] *Institut de Physique Théorique, CEA-Saclay, F-91191 Gif-sur-Yvette Cedex, France* *

Abstract

We study flavour violation in a supersymmetric SO(10) implementation of the type II seesaw mechanism, which provides a predictive realization of triplet leptogenesis. The experimental upper bounds on lepton flavour violating processes have a significant impact on the leptogenesis dynamics, in particular they exclude the strong washout regime. Requiring successful leptogenesis then constrains the otherwise largely unknown overall size of flavour-violating observables, thus yielding testable predictions. In particular, the branching ratio for $\mu \to e\gamma$ lies within the reach of the MEG experiment if the superpartner spectrum is accessible at the LHC, and the supersymmetric contribution to ε_K can account for a significant part of the experimental value. We show that this scenario can be realized in a consistent SO(10) model achieving gauge symmetry breaking and doublet-triplet splitting in agreement with the proton decay bounds, improving on the MSSM prediction for $\alpha_3(m_Z)$, and reproducing the measured quark and lepton masses.

*Laboratoire de la Direction des Sciences de la Matière du Commissariat à l'Energie Atomique et Unité de Recherche Associée au CNRS (URA 2306).

1 Introduction

Neutrino masses presumably arise at a scale much larger than the electroweak scale. If this is the case, a model-independent effective description of neutrino masses is possible at lower scales in terms of the dimension 5 operator $l_i l_j h_u h_u / \Lambda$ [1], where l_i is the i-th family lepton doublet, h_u is the $Y = +1/2$ Higgs doublet and Λ is the scale at which the operator is generated, which can be as large as 10^{15} GeV. While this effective description is essentially unique, the high-energy mechanism leading to the above dimension 5 operator is not. Generally, it is assumed to arise from the tree-level exchange of SM singlet fermions (type I seesaw mechanism [2]). In this case, the low-energy information from lepton masses and mixing only determines 9 out of the 18 high-energy seesaw parameters. Due to this arbitrariness, it is not possible to make definite predictions for the lepton asymmetry generated in the decays of the heavy singlet neutrinos [3], nor, in supersymmetric models, for the lepton flavour violating (LFV) effects induced by their Yukawa interactions [4].

On the other hand, the exchange of singlet fermions is not the only possible origin of the $l_i l_j h_u h_u / \Lambda$ operator. In this paper, we consider the exchange of an $\mathrm{SU}(2)_\mathrm{L}$ triplet scalar with $Y = \pm 1$ (type II or triplet seesaw mechanism [5]). More precisely, we consider the SO(10) [6] implementation of the triplet seesaw mechanism proposed in Ref. [7], in which all flavour parameters contributing to low-energy observables are determined in terms of the SM fermion masses and mixings[1]. This allows to make testable predictions for these observables, up to a few unknown flavour-blind parameters. Particularly interesting is the possibility [7] of accounting for the baryon asymmetry of the universe via triplet leptogenesis. In usual type II models, it is necessary to introduce additional triplets or singlets in order to provide a rich enough flavour structure to induce a non-vanishing CP asymmetry in triplet decays [8]. This brings back into the game a number of flavour unknowns. On the contrary, in the scenario that we consider, the additional states are heavy quarks and leptons whose masses and couplings are determined in terms of low-energy parameters through SO(10) relations. The generated baryon asymmetry then directly depends on the light neutrino parameters.

In this paper, we pursue the exploration of this scenario by studying the flavour- and CP-violating effects induced by the couplings of the heavy states to the MSSM squarks and sleptons. Assuming flavour-universal soft supersymmetry breaking terms at the GUT scale, flavour-violating observables are predicted up to a few unknown scale parameters and to a mild model-dependent uncertainty, thus allowing to test the scenario. Besides the contributions of the type II seesaw triplet and of its SU(5) partners already studied in Ref. [9], the presence of heavy quarks and leptons gives rise to additional contributions to the slepton and squark soft terms. In particular, they induce flavour and CP violation in the slepton singlet and squark doublet sectors, which were absent in the SU(5) type II seesaw model.

The paper is organized as follows. In Section 2, we present the main features of the SO(10) scenario we consider. Section 3 contains a qualitative discussion of its predictions for flavour and CP violation. Model-building aspects including gauge coupling unification, doublet-triplet splitting and proton decay are briefly discussed in Section 4, and addressed in greater detail in Appendices A, B and C, where the main ingredients of a realistic model are given. Section 5 presents our numerical results. Finally, we give our conclusions in Section 6. The superpotential

[1] Actually, the higher-dimensional operators needed to account for the measured masses of down quarks and charged leptons may affect this relation between high-energy and low-energy flavour parameters. We argue in Section 3.3 that the impact of this on physical observables is generally small, and neglect it in the following.

of the model, the boundary conditions for Yukawa couplings and soft terms and a subset of the renormalization group equations are displayed in Appendix D.

2 SO(10) unification with type II seesaw mechanism

We consider a supersymmetric SO(10) scenario with matter fields in **16** and **10** representations and type II realization of the seesaw mechanism, in which soft supersymmetry breaking terms arise at the GUT scale or higher. Since we are interested in the flavour-violating effects induced by the physics responsible for neutrino masses, we assume that the soft terms are flavour universal at the GUT scale.

Let us first describe the field content. In terms of SU(5) multiplets, the three families of SM fermions are described by $\bar{\mathbf{5}}_i^{\mathrm{SU}(5)} \oplus \mathbf{10}_i^{\mathrm{SU}(5)}$, $i = 1, 2, 3$. In conventional SO(10) unification [6], $\bar{\mathbf{5}}_i^{\mathrm{SU}(5)}$ and $\mathbf{10}_i^{\mathrm{SU}(5)}$ are unified in a single $\mathbf{16}_i^{\mathrm{SO}(10)}$ together with a singlet (right-handed) neutrino participating in the generation of light neutrino masses through the type I seesaw mechanism. In this paper, we consider the alternative possibility that $\mathbf{10}_i^{\mathrm{SU}(5)}$ is embedded in a $\mathbf{16}_i^{\mathrm{SO}(10)}$, while $\bar{\mathbf{5}}_i^{\mathrm{SU}(5)}$ belongs to a $\mathbf{10}_i^{\mathrm{SO}(10)}$ (for similar or related approaches, see Ref. [10]). Having done this choice, we drop from now on the "SO(10)" superscript on SO(10) representations, and indicate the embedding of SU(5) representations into SO(10) representations by a superscript: for instance, $\bar{\mathbf{5}}_i^{\mathbf{10}}$ is contained in $\mathbf{10}_i$, while $\bar{\mathbf{5}}_i^{\mathbf{16}}$ belongs to $\mathbf{16}_i$. Besides the three $\mathbf{16}_i$ and $\mathbf{10}_i$ matter multiplets, the model also involves a **10**, a **16** and a **54** Higgs multiplets. These are needed in order to generate the quark and charged lepton masses, as well as the neutrino masses through the type II seesaw mechanism. The corresponding SO(10) superpotential reads:

$$
\begin{aligned}
W &= W_{\mathrm{Yukawa}} + W_{\mathrm{seesaw}} + W_{\mathrm{GUT}} + W_{\mathrm{nonren.}}\,, && (1)\\
W_{\mathrm{Yukawa}} &= \frac{1}{2}\, y_{ij} \mathbf{16}_i \mathbf{16}_j \mathbf{10} + h_{ij} \mathbf{16}_i \mathbf{10}_j \mathbf{16}\,, && (2)\\
W_{\mathrm{seesaw}} &= \frac{1}{2}\, f_{ij} \mathbf{10}_i \mathbf{10}_j \mathbf{54} + \frac{1}{2}\, \sigma \mathbf{10}\,\mathbf{10}\,\mathbf{54} + \frac{1}{2}\, M_{54} \mathbf{54}^2\,, && (3)
\end{aligned}
$$

where W_{GUT}, whose explicit form is given in Appendices A and B, contains the terms responsible for SO(10) symmetry breaking and for the doublet-triplet splitting, and $W_{\mathrm{nonren.}}$ includes the non-renormalizable operators needed to account for the measured ratios of down quark and charged lepton masses. The role of the **10**, **16** and **54** representations is the following. The **10** contains the $Y = +1/2$ MSSM Higgs doublet h_u responsible for up quark masses. The **16** plays a double role: it shares the $Y = -1/2$ MSSM Higgs doublet h_d with the **10** and (together with its $\overline{\mathbf{16}}$ companion) it reduces the rank of the unified gauge group down to 4 through the vev of its SU(5)-singlet component; moreover, this vev pairs up the spare $\bar{\mathbf{5}}_i^{\mathbf{16}}$ and $\mathbf{5}_i^{\mathbf{10}}$ and gives them a large mass, leaving only the $\bar{\mathbf{5}}_i^{\mathbf{10}}$ massless. The **54** contains the $\mathrm{SU}(2)_{\mathrm{L}}$ triplet mediating the type II seesaw mechanism. The three singlet neutrinos contained in the $\mathbf{16}_i$, on the other hand, do not couple to the light neutrinos at the renormalizable level. Assuming for definiteness that they acquire GUT-scale masses (e.g. from couplings $\mathbf{1}_i \mathbf{16}_j \overline{\mathbf{16}}$ to three SO(10) singlets $\mathbf{1}_i$), we can completely neglect their contributions to the light neutrino masses, if any, and we are left with a pure type II seesaw mechanism.

In writing Eqs. (2) and (3), we assumed that the superpotential is invariant under a matter parity, with the matter fields being the $\mathbf{16}_i$ and the $\mathbf{10}_i$. We also required the absence of mass terms of the form $\mathbf{10}_i \mathbf{10}_j$ and of additional interactions that would induce a vev for the **54**, in order to prevent a mixing between the $\bar{\mathbf{5}}_i^{\mathbf{10}}$ and the $\bar{\mathbf{5}}_i^{\mathbf{16}}$. The absence of such a mixing

ensures that the light neutrino masses arise from a pure type II seesaw mechanism, and is a crucial feature of the scenario. The scale of light neutrino masses requires M_{54} to be smaller than the GUT scale; it is therefore natural to assume that this mass term arises at the non-renormalizable level, while it is forbidden or small at the renormalizable level. Furthermore, the leptogenesis scenario proposed in Ref. [7] requires that the $\mathbf{24^{54}}$ component of the $\mathbf{54}$ be heavier than the $(\mathbf{15} \oplus \overline{\mathbf{15}})^{\mathbf{54}}$ one. We refer the reader to Appendix C for the discussion of the splitting of the $\mathbf{54}$.

The degrees of freedom surviving below the GUT scale are the MSSM states, which are massless before electroweak symmetry breaking, three heavy pairs of vector-like matter $\bar{\mathbf{5}}_i^{\mathbf{16}} \oplus \mathbf{5}_i^{\mathbf{10}}$, and the components of the $\mathbf{54}$, which are also heavy. The light MSSM fields and the heavy ones are embedded in the SO(10) representations of the model as follows:

$$\begin{aligned}
\mathbf{16}_i &= \mathbf{10}_i^{\mathbf{16}} \oplus \bar{\mathbf{5}}_i^{\mathbf{16}} \oplus \mathbf{1}_i^{\mathbf{16}} && = (q_i, u_i^c, e_i^c) \oplus (D_i^c, L_i) \oplus n_i^c \ , \\
\mathbf{10}_i &= \bar{\mathbf{5}}_i^{\mathbf{10}} \oplus \mathbf{5}_i^{\mathbf{10}} && = (d_i^c, l_i) \oplus (\bar{D}_i^c, \bar{L}_i) \ , \\
\mathbf{54} &= \mathbf{24^{54}} \oplus \mathbf{15^{54}} \oplus \overline{\mathbf{15}}^{\mathbf{54}} && = (S, T, O, V, \bar{V}) \oplus (\Delta, \Sigma, Z) \oplus (\bar{\Delta}, \bar{\Sigma}, \bar{Z}) \ , \\
\mathbf{10} &= \bar{\mathbf{5}}^{\mathbf{10}} \oplus \mathbf{5}^{\mathbf{10}} && = (\cdots, \cos\theta_H h_d) \oplus (\cdots, h_u) \ , \\
\mathbf{16} &= \mathbf{10^{16}} \oplus \bar{\mathbf{5}}^{\mathbf{16}} \oplus \mathbf{1^{16}} && = \cdots \oplus (\cdots, \sin\theta_H h_d) \oplus \cdots \ ,
\end{aligned} \tag{5}$$

where dots stands for heavy fields which have been integrated out at the GUT scale (this includes in particular the coloured Higgs triplets mediating $D = 5$ proton decay). We have introduced an angle θ_H parametrizing the mixing among $Y = -1/2$ Higgs doublets ($0 < \theta_H < \pi/2$ with no loss of generality) and assumed that the $Y = +1/2$ MSSM Higgs doublet entirely resides in the $\mathbf{10}$, so that all components of the $\overline{\mathbf{16}}$ are heavy (see Appendix B for an explicit realization of this). The $SU(3)_C \times SU(2)_L \times U(1)_Y$ quantum numbers of the $\mathbf{54}$ components are $\Delta = (1,3)_{+1}$, $\Sigma = (6,1)_{-2/3}$, $Z = (3,2)_{+1/6}$, $S = (1,1)_0$, $T = (1,3)_0$, $O = (8,1)_0$ and $V \sim (3,2)_{-5/6}$, while the heavy L_i and D_i^c obviously carry the same quantum numbers as the light l_i and d_i^c. After breaking of the SO(10) gauge symmetry, the heavy quark and lepton fields acquire Dirac masses $(M_{D^c})_{ij} D_i^c \bar{D}_j^c + (M_L)_{ij} L_i \bar{L}_j$, where, at the renormalizable level and neglecting the renormalization group running below the GUT scale:

$$(M_{D^c})_{ij} \;=\; (M_L)_{ij} \;=\; h_{ij} V_1 \ , \tag{7}$$

in which V_1 is the vev of the SU(5)-singlet component of the $\mathbf{16}$. As for the SM fermion masses, they are given by (again neglecting corrections from non-renormalizable operators and the RG running of the Yukawa couplings):

$$\begin{aligned}
(M_u)_{ij} &= y_{ij} v_u \ , \\
(M_e)_{ij} &= \sin\theta_H h_{ij} v_d \ , \\
(M_d)_{ij} &= \sin\theta_H h_{ij}^T v_d \ ,
\end{aligned} \tag{8}$$

and the neutrino masses are given by the type II seesaw formula:

$$(M_\nu)_{ij} \;=\; -\frac{\sigma v_u^2}{2 M_\Delta} f_{ij} \ . \tag{9}$$

Note that the down quark and charged lepton masses, which satisfy the SU(5) relation $M_d = M_e^T$ at the renormalizable level, are proportional to $\sin\theta_H$, the $Y = -1/2$ Higgs mixing

parameter[2]. Due to the way the SM fermions are embedded into SO(10) representations, the up quark mass matrix is not correlated with the down quark and charged lepton mass matrices as in conventional SO(10) models, which accounts in a natural way for the stronger mass hierarchy observed in the up quark sector.

The role of the SO(10) symmetry is to relate the masses and couplings of the heavy matter fields $(L_i, \bar{L}_i)$ and $(D_i^c, \bar{D}_i^c)$ to the ones of the light fermions, thus allowing to predict their contributions to observables such as the baryon asymmetry of the universe and the rates of flavour-violating processes. Indeed, Eqs. (2)–(3) and (7)–(9) show that all flavour parameters involved in the superpotential, as well as the masses of the heavy matter fields, are determined by the SM fermion masses and mixings, up to flavour-blind factors and to the non-renormalizable contributions needed to make $M_d \neq M_e^T$. On the contrary, in conventional SO(10) models where neutrino masses arise from the type I seesaw mechanism, the Lagrangian below the GUT scale also depends on the flavour parameters encoded in the so-called R matrix [11]. The SO(10) scenario studied in this paper therefore has a higher predictive power. As for the unknowns associated with the non-renormalizable operators in $W_{\text{nonren.}}$, we will argue in Section 3.3 that they are unlikely to affect our results in a sizable way.

3 Flavour and CP violation

3.1 General structure of radiative corrections to the MSSM soft terms

The $f_{ij}\mathbf{10}_i\mathbf{10}_j\mathbf{54}$ interactions introduce a new flavour structure, directly related to the light neutrino masses, on top of the MSSM one. As noted in Ref. [12], this can be considered as a truly minimal extension to the lepton sector of the minimal flavour violation hypothesis in the quark sector, which does not rely on flavour basis dependent assumptions. In any case, the $f_{ij}\mathbf{10}_i\mathbf{10}_j\mathbf{54}$ interactions give rise to new flavour- and CP-violating effects at low energy because of their well-known impact on the MSSM soft terms through radiative corrections [9]. In addition, the $y_{ij}\mathbf{16}_i\mathbf{16}_j\mathbf{10}$ interactions induce new flavour and CP violation in the slepton singlet sector due to the presence of heavy lepton doublets in the $\mathbf{16}_i$, as we are going to see.

In order to compute these effects, we must write the renormalisation group equations (RGEs) for the MSSM parameters and integrate them. Before doing so, let us note that the l_i and L_i fields can mix, since they have the same quantum numbers after breaking of the SO(10) symmetry (and similarly for the d_i^c and D_i^c fields). Hence, the superpotential mass term for the lepton doublets reads, in full generality, $L_i(M_L)_{ij}\bar{L}_j + l_i(M_{lL})_{ij}\bar{L}_j$. The second term was omitted previously since M_{lL} vanishes at the tree level, while M_L is given by Eq. (7). At one loop, however, wave function renormalization induces a small l_i-L_j mixing:

$$(M_{lL})_{ij} \ \approx\ -\frac{\sin 2\theta_H}{32\pi^2}\, (M_L y^\dagger h)_{ij} \ln\left(\frac{M_{\text{GUT}}}{\mu}\right) . \tag{10}$$

This in turn necessitates a redefinition of the heavy and light lepton doublets, which affects all couplings involving lepton doublets. However, the size of the effect is small (especially for small or moderate $\tan\beta$), and we shall neglect it in the following.

Before solving numerically the full 1-loop RGEs, let us illustrate the main features of the results by using the leading-log approximation for the exchange of the heavy degrees of freedom.

[2]This offers the possibility of explaining the smallness of the bottom/top mass hierarchy for moderate values of $\tan\beta$ in terms of a small Higgs mixing angle θ_H. For reasons explained in Section 4.1, however, we shall not use this possibility here.

In order to be able to identify each contribution, we assume that all components of the **54**, namely S, T, O and the vector-like pairs $(V, \bar{V})$ $(\Delta, \bar{\Delta})$, $(\Sigma, \bar{\Sigma})$ and $(Z, \bar{Z})$ have different masses (the mass M_Z of the $(Z,\, \bar{Z})$ vector-like pair should not be confused with the Z boson mass m_Z). We also consider the possibility that $M_{D^c} \neq M_L$, but ignore the effect of corrections to the mass relation $M_d = M_e^T$. Let us denote by m_{16}^2 and m_{10}^2 the universal soft terms for the three families of $\mathbf{16}_i$ and $\mathbf{10}_i$ matter fields at the GUT scale; by m_{54}, $m_{16_H}^2$ and $m_{10_H}^2$ the soft terms for the **54**, **16** and **10** SO(10) multiplets; and by a_0 the universal A-term (defined by $A_x = a_0 x$, where x is any superpotential trilinear coupling x). Using the RGEs given in Appendix D, we obtain (in matrix form):

$$\begin{aligned}
m_l^2 \;&=\; (m_l^2)_{\rm MSSM} - \frac{1}{(4\pi)^2}\left(2m_{10}^2 + m_{54}^2 + a_0^2\right) f^\dagger \left[\frac{3}{2}\ln\frac{M_{\rm GUT}^2}{M_\Delta^2} + \frac{3}{2}\ln\frac{M_{\rm GUT}^2}{M_Z^2}\right. \\
&\left. + \frac{3}{4}\ln\frac{M_{\rm GUT}^2}{M_T^2 + M_L^T M_L^*} + \frac{3}{20}\ln\frac{M_{\rm GUT}^2}{M_S^2 + M_L^T M_L^*} + \frac{3}{2}\ln\frac{M_{\rm GUT}^2}{M_V^2 + M_{D^c}^T M_{D^c}^*}\right] f \;, \qquad \text{(11a)} \\
m_{d^c}^2 \;&=\; (m_{d^c}^2)_{\rm MSSM} - \frac{1}{(4\pi)^2}\left(2m_{10}^2 + m_{54}^2 + a_0^2\right) f \left[2\ln\frac{M_{\rm GUT}^2}{M_\Sigma^2} + \ln\frac{M_{\rm GUT}^2}{M_Z^2}\right. \\
&\left. + \ln\frac{M_{\rm GUT}^2}{M_V^2 + M_L^\dagger M_L} + \frac{4}{3}\ln\frac{M_{\rm GUT}^2}{M_O^2 + M_{D^c}^\dagger M_{D^c}} + \frac{1}{15}\ln\frac{M_{\rm GUT}^2}{M_S^2 + M_{D^c}^\dagger M_{D^c}}\right] f^\dagger \;. \qquad \text{(11b)}
\end{aligned}$$

In contrast to the well-known type I seesaw case, the flavour-violating corrections to m_l^2 are determined by the light neutrino mass matrix, with no ambiguity due to high-energy flavour parameters. Therefore, while the absolute rate of a given LFV process is not known, the correlations between different LFV channels are predicted with little uncertainty (at least if the contributions of the slepton singlet sector, to be discussed below, are subdominant). The first term in the squared brackets of Eq. (11a) is induced by the seesaw triplet interactions and is present in all type II seesaw models. The second term, as well as the first two terms in the squared brackets of Eq. (11b), corresponds to the contribution of the SU(5) partners of the triplet [9]. The additional terms in m_l^2 and $m_{d^c}^2$ are due to the presence of heavy leptons and quarks in the $\mathbf{16}_i$, and are characteristic of the model studied in this paper. In the limit $M_L = M_{D^c}$, $M_\Delta = M_\Sigma = M_Z$, $M_S = M_T = M_O = M_V$, the correction to the MSSM evolution is the same for m_l^2 and $(m_{d^c}^2)^T$, as dictated by the SU(5) invariance of the interactions from which they arise.

Another difference from other type II seesaw models comes from the corrections to the MSSM running of $m_{e^c}^2$ and m_q^2:

$$\begin{aligned}
m_{e^c}^2 \;&=\; (m_{e^c}^2)_{\rm MSSM} - \frac{\cos^2\theta_H}{(4\pi)^2}\left(2m_{16}^2 + m_{h_d}^2 + a_0^2\right) y \left[2\ln\frac{M_{\rm GUT}^2}{M_L^* M_L^T}\right] y^\dagger \;, \qquad \text{(12a)} \\
m_q^2 \;&=\; (m_q^2)_{\rm MSSM} - \frac{\cos^2\theta_H}{(4\pi)^2}\left(2m_{16}^2 + m_{h_d}^2 + a_0^2\right) y^\dagger \left[\ln\frac{M_{\rm GUT}^2}{M_{D^c} M_{D^c}^\dagger}\right] y \;, \qquad \text{(12b)}
\end{aligned}$$

where $m_{h_d}^2 = \cos^2\!\theta_H\, m_{10_H}^2 + \sin^2\!\theta_H\, m_{16_H}^2$. These are controlled by the up quark Yukawa couplings (evolved at the high scale). While the corrections to m_q^2 do not represent a deviation from the minimal flavour violation structure of the MSSM radiative corrections, the corrections to $m_{e^c}^2$ are similar to the ones induced above the GUT scale by the top quark Yukawa coupling in SU(5) models [13], although their origin is different.

3.2 Analytic approximations for the mass insertions and CP violation

In order to be able to quickly estimate the size of various flavour- and CP-violating observables, it is useful to provide analytic expressions for the mass insertions parameters ($i \neq j$) [14]:

$$(\delta^e_{LL})_{ij} \equiv \frac{(m^2_l)_{ij}}{\overline{m}^2_{\tilde{e}_L}} , \quad (\delta^e_{RR})_{ij} \equiv \frac{(m^2_{e^c})_{ij}}{\overline{m}^2_{\tilde{e}_R}} , \quad (\delta^e_{RL})_{ij} \equiv \frac{(A_e)_{ij} v_d}{\overline{m}_{\tilde{e}_L} \overline{m}_{\tilde{e}_R}} , \tag{13}$$

where $\overline{m}_{\tilde{e}_L}$ ($\overline{m}_{\tilde{e}_R}$) is an average doublet (singlet) slepton mass, and analogous quantities are defined in the up and down squark sectors. These parameters can be straightforwardly derived from Eqs. (11) and (12), but some care is needed regarding CP-violating phases. Even if we take real boundary conditions for the soft supersymmetry breaking parameters at $M_{\rm GUT}$, we will end up with complex soft terms at the weak scale because their RGEs involve complex couplings. These couplings contain, in addition to the CKM and PMNS phases, extra CP-violating phases inherited from the SO(10) structure. In order to identify the latter, it is useful to write the SO(10) Yukawa couplings in an appropriate basis for the SO(10) matter multiplets $\mathbf{16}_i$ and $\mathbf{10}_i$, namely[3]:

$$y = U_q^T D_y U_q , \qquad h = D_h , \qquad f = U^* D_f U^\dagger , \tag{14}$$

where

$$U_q = \mathrm{Diag}(e^{i\Phi^u_1}, e^{i\Phi^u_2}, e^{i\Phi^u_3})\, V\, \mathrm{Diag}(e^{i\Phi^d_1}, e^{i\Phi^d_2}, e^{i\Phi^d_3}) . \tag{15}$$

In Eqs. (14) and (15), D_y, D_h, D_f are real diagonal matrices, U and V are the PMNS and CKM matrices in the standard PDG parametrization (extended to include two "Majorana" phases ρ and σ in the PMNS case), and Φ^u_i, Φ^d_i ($i = 1, 2, 3$) are extra SO(10) phases, five of which are independent (one can impose e.g. $\Phi^u_3 = 0$). For simplicity, we neglect the effects of the non-renormalizable operators needed to correct the mass relation $M_d = M_e^T$, and we therefore also assume $M_{D^c} = M_L = D_h V_1$. Below the GUT scale, we are free to rephase independently the MSSM fields so that the Yukawa couplings $\lambda_{u,d,e}$ and f_Δ only contain CKM and PMNS phases; however, the extra SO(10) phases will reappear in other couplings, such as $\hat{\lambda}_d$ and f_Z, which enter the RGEs for the MSSM soft terms (we refer to Appendix D for the definition of the superpotential couplings below the GUT scale). The outcome of this is that the mass insertion parameters depend on the phase differences $\Phi^u_i - \Phi^u_j$ and $\Phi^d_i - \Phi^d_j$ in addition to the 4 low-energy phases δ_{CKM}, δ_{PMNS}, ρ and σ.

We are now ready to write the mass insertion parameters $(\delta^{e,d,u}_{MN})_{ij}$ ($M, N = L, R$) in the leading-log approximation, assuming for simplicity $M_\Delta = M_\Sigma = M_Z \equiv M_{15}$, $M_S = M_T = M_O = M_V \equiv M_{24}$, a common soft supersymmetry breaking mass m_0 for all SO(10) chiral multiplets and, as before, a common A-terms a_0 for all superpotential trilinear couplings. We obtain:

$$16\pi^2 (\delta^e_{LL})_{ij} \approx -\frac{3m_0^2 + |a_0|^2}{\overline{m}^2_{\tilde{e}_L}}\, C^f_{ij} , \tag{16}$$

$$16\pi^2 (\delta^e_{RR})_{ij} \approx -\frac{2(3m_0^2 + |a_0|^2)}{\overline{m}^2_{\tilde{e}_R}}\, \mathrm{e}^{i(\Phi^d_i - \Phi^d_j)}\, (C^{\lambda_u}_{ij})^* , \tag{17}$$

$$16\pi^2 (\delta^e_{RL})_{ij} \approx -\frac{3a_0}{2\overline{m}_{\tilde{e}_L}\overline{m}_{\tilde{e}_R}} \left[m_{e_i} C^f_{ij} + 3\, \mathrm{e}^{i(\Phi^d_i - \Phi^d_j)}\, (C^{\lambda_u}_{ij})^* m_{e_j} \right] , \tag{18}$$

[3] In the following discussion, we neglect the effect of radiative corrections in the Yukawa sector. This allows us to disentangle the CKM and PMNS phases from the extra SO(10) phases, which would otherwise be mixed by the renormalization group running.

$$16\pi^2(\delta^d_{LL})_{ij} \;\approx\; -\frac{3m_0^2+|a_0|^2}{\overline{m}^2_{\tilde{d}_L}} \left[(C^{\lambda_u}_{\rm MSSM})_{ij} + C^{\lambda_u}_{ij} \right], \tag{19}$$

$$16\pi^2(\delta^d_{RR})_{ij} \;\approx\; -\frac{3m_0^2+|a_0|^2}{\overline{m}^2_{\tilde{d}_R}}\, {\rm e}^{i(\Phi^d_i-\Phi^d_j)}\, (C^f_{ij})^*, \tag{20}$$

$$16\pi^2(\delta^d_{RL})_{ij} \;\approx\; -\frac{3a_0}{2\overline{m}_{\tilde{d}_L}\overline{m}_{\tilde{d}_R}} \left\{ m_{d_i} \left[(C^{\lambda_u}_{\rm MSSM})_{ij} + 3\, C^{\lambda_u}_{ij} \right] + {\rm e}^{i(\Phi^d_i-\Phi^d_j)}\, (C^f_{ij})^* m_{d_j} \right\}, \tag{21}$$

$$16\pi^2(\delta^u_{LL})_{ij} \;\approx\; -\frac{3m_0^2+|a_0|^2}{\overline{m}^2_{\tilde{u}_L}} \left[(C^{\lambda_d}_{\rm MSSM})_{ij} + {\rm e}^{-2i(\Phi^u_i-\Phi^u_j)}\, D^{\lambda_u}_{ij} \right], \tag{22}$$

$$16\pi^2(\delta^u_{RR})_{ij} \;\approx\; 0\ , \tag{23}$$

$$16\pi^2(\delta^u_{RL})_{ij} \;\approx\; -\frac{3a_0}{2\overline{m}_{\tilde{u}_L}\overline{m}_{\tilde{u}_R}}\, m_{u_i} \left[(C^{\lambda_d}_{\rm MSSM})_{ij} + {\rm e}^{-2i(\Phi^u_i-\Phi^u_j)}\, D^{\lambda_u}_{ij} \right], \tag{24}$$

where the dependence on the low-energy flavour parameters and phases is encapsulated in the coefficients $(C^{\lambda_u}_{\rm MSSM})_{ij}$, $(C^{\lambda_d}_{\rm MSSM})_{ij}$, C^f_{ij}, $C^{\lambda_u}_{ij}$ and $D^{\lambda_u}_{ij}$. The first two correspond to the CKM-induced MSSM contributions and are given by:

$$(C^{\lambda_u}_{\rm MSSM})_{ij} \;\simeq\; \lambda_t^2\, V^*_{ti} V_{tj}\, \ln\left(\frac{M^2_{\rm GUT}}{m^2_Z}\right), \quad (C^{\lambda_d}_{\rm MSSM})_{ij} \;=\; \sum_k \lambda^2_{d_k} V_{ik} V^*_{jk}\, \ln\left(\frac{M^2_{\rm GUT}}{m^2_Z}\right), \tag{25}$$

where terms suppressed by λ_c/λ_t have been neglected in $(C^{\lambda_u}_{\rm MSSM})_{ij}$. The other three coefficients correspond to the contributions of the heavy states present below the GUT scale and also contain a dependence on their masses. The one that involves the seesaw couplings is given by:

$$\begin{aligned} C^f_{ij} \;&=\; 3\,(f^\dagger f)_{ij}\, \ln\left(\frac{M^2_{\rm GUT}}{M^2_{15}}\right) + \frac{12}{5} \sum_a f^*_{ai} f_{aj}\, \ln\left(\frac{M^2_{\rm GUT}}{M^2_{24}+M^2_{5_a}}\right) \\ &=\; 3 \sum_k f^2_k\, U_{ik} U^*_{jk}\, \ln\left(\frac{M^2_{\rm GUT}}{M^2_{15}}\right) + \frac{12}{5} \sum_{k,l} f_k f_l\, U_{ik} U^*_{jl} \sum_a U_{ak} U^*_{al}\, \ln\left(\frac{M^2_{\rm GUT}}{M^2_{24}+M^2_{5_a}}\right), \end{aligned} \tag{26}$$

where $f_i = (2M_\Delta/\sigma v_u^2)\, m_{\nu_i}$ and the M_{5_a} $(a=1,2,3)$ are the common masses of the heavy quarks and leptons, and the coefficients that involve the up quark Yukawa couplings read:

$$C^{\lambda_u}_{ij} \;\simeq\; \cos^2\theta_H\, \lambda_t^2\, V^*_{ti} V_{tj}\, \ln\left(\frac{M^2_{\rm GUT}}{M^2_{5_3}}\right), \quad D^{\lambda_u}_{ij} \;=\; \cos^2\theta_H\, \lambda_{u_i} \lambda_{u_j} \sum_a V^*_{ia} V_{ja}\, \ln\left(\frac{M^2_{\rm GUT}}{M^2_{5_a}}\right), \tag{27}$$

where terms suppressed by λ_c/λ_t have been neglected in $C^{\lambda_u}_{ij}$. Due to this (very good) approximation, only the CKM and PMNS phases appear in Eqs. (25) to (27).

Let us have a closer look at the mass insertion parameters (18) to (24). In the up squark sector, the new contributions are subdominant with respect to the MSSM corrections, which are enhanced by $\tan^2\beta$ and by a large logarithm. The situation is more interesting in the down squark and charged slepton sectors. In the former, the new corrections controlled by the top quark Yukawa coupling are again subdominant with respect to the MSSM corrections, while they lead to RR flavour violation in the latter. The new corrections induced by the seesaw couplings, on the other hand, are not suppressed by small CKM angles and can give rise to large flavour violations in the RR down squark sector and in the LL slepton sector [9]. One observes the following correlations:

$$\overline{m}^2_{\tilde{d}_R} (\delta^d_{RR})_{ij} \;=\; \overline{m}^2_{\tilde{e}_L} (\delta^e_{LL})^*_{ij}\, {\rm e}^{i(\Phi^d_i-\Phi^d_j)}\ , \tag{28}$$

$$\overline{m}^2_{\tilde{e}_R} (\delta^e_{RR})_{ij} \;=\; 2R\, \overline{m}^2_{\tilde{d}_L} (\delta^d_{LL})^*_{ij}\, {\rm e}^{i(\Phi^d_i-\Phi^d_j)}\ , \tag{29}$$

$\sin^2\theta_{13}$ $\tan\beta$	0.05 10	0 10	0.05 50	0 50
$\lvert\delta^{LL}_{e\mu}\rvert$	2.0×10^{-3}	1.7×10^{-4}	2.0×10^{-3}	1.5×10^{-4}
$\lvert\delta^{LL}_{e\tau}\rvert$	1.8×10^{-3}	1.0×10^{-4}	1.9×10^{-3}	1.4×10^{-4}
$\lvert\delta^{LL}_{\mu\tau}\rvert$	5.7×10^{-3}	5.9×10^{-3}	6.1×10^{-3}	6.3×10^{-3}
$\lvert\delta^{RR}_{e\mu}\rvert$	2.1×10^{-6}	2.1×10^{-6}	2.8×10^{-8}	4.6×10^{-7}
$\lvert\delta^{RR}_{e\tau}\rvert$	5.8×10^{-5}	5.4×10^{-5}	1.4×10^{-4}	1.2×10^{-6}
$\lvert\delta^{RR}_{\mu\tau}\rvert$	4.3×10^{-4}	4.3×10^{-4}	2.9×10^{-4}	2.9×10^{-4}

Table 1: Numerical values of the leptonic mass insertion parameters δ^e_{LL}, δ^e_{RR} for two different values of $\sin^2\theta_{13}$ and $\tan\beta$ and for the choice of parameters mentioned in the text.

where $R \equiv \cos^2\theta_H \ln(M^2_{\rm GUT}/M^2_{5_3}) / \left[\ln(M^2_{\rm GUT}/m^2_Z) + \cos^2\theta_H \ln(M^2_{\rm GUT}/M^2_{5_3})\right]$. The first one is characteristic of the SU(5) extension of the type II seesaw mechanism, while the second one is analogous (although from a different origin) to the one arising from the running between M_P and $M_{\rm GUT}$ in the minimal SU(5) model. In the case that the seesaw-induced corrections dominate, one further has:

$$(\delta^e_{RR})_{ij} \ll (\delta^e_{LL})_{ij} , \qquad (\delta^d_{LL})_{ij} \ll (\delta^d_{RR})_{ij} , \tag{30}$$

$$\overline{m}_{\tilde{d}_R}\overline{m}_{\tilde{d}_L}(\delta^d_{RL})_{ij} = \overline{m}_{\tilde{e}_R}\overline{m}_{\tilde{e}_L}(\delta^e_{RL})_{ji}\, \mathrm{e}^{i(\Phi^d_i-\Phi^d_j)} . \tag{31}$$

Eqs. (28), (29) and (31) correlate the size of flavour-changing neutral currents (FCNCs) in the lepton and B/K sectors, as well as the charged lepton and quark electric dipole moments (EDMs), as we are going to see.

For illustration, we give in Table 1 numerical values for the leptonic δ's. These values were obtained by numerically solving the RGEs for the following choice of high-energy parameters, motivated by the leptogenesis scenario of Ref. [7]: $M_\Delta = M_\Sigma = M_Z \equiv M_{15} = 10^{12}\,$GeV, $M_S = M_T = M_O = M_V \equiv M_{24} = 10^{13}\,$GeV, $\sigma = 3.4 \times 10^{-2}$, $V_1 = M_{\rm GUT} = 2 \times 10^{16}\,$GeV and $\tan\theta_H = 1$. For the heavy $(\mathbf{5}_i, \bar{\mathbf{5}}_i)$ masses, we used $M_{5_i} = m_{e_i} V_1 / \sin\theta_H v_d$, neglecting SU(5)-breaking corrections. The supersymmetric parameters were chosen to be $m_0 = M_{1/2} = 1.5\,$TeV, $a_0 = 0$, and $\tan\beta = 10$ (50), yielding the following superpartner masses (for $\tan\beta = 10$): $M_{\tilde\chi^0_1} = 112\,$GeV, $M_{\tilde\chi^+_1} = 218\,$GeV, $M_{\tilde g} = 723\,$GeV, sleptons around $(1.5-1.6)\,$TeV and first two generation squarks around $1.8\,$TeV (as well as $\mu = 562\,$GeV). Finally, we took the best fit values of Ref. [15] for the measured neutrino oscillation parameters, together with $m_1 = 0.005\,$eV and $\sin^2\theta_{13} = 0$ (0.05) for the yet unknown parameters. All phases were set to zero. The resulting values of the leptonic mass insertions shown in Table 1 can be compared with the bounds coming from the non-observation of LFV decays of charged leptons (see e.g. Refs. [16, 17, 18]). Although the sleptons are heavy in this example, the relatively large values of the δ's lead to a branching ratio for $\mu \to e\gamma$ just below the experimental bound for $\tan\beta = 10$, and above it for $\tan\beta = 50$. Using Eqs. (28), (29) and (31), one can also compare the figures in Table 1 with the constraints on hadronic δ's coming from B and K physics (see e.g. Refs. [19, 18]).

From Table 1 we can see that the LL mass insertions are the dominant source of lepton flavour violation in our numerical example, i.e. we are in the situation described by Eq. (30). This is due to the fact that the choice made for the model parameters leads to rather large f_{ij}'s (namely $f_{33} \sim 0.1$): a smaller value of the ratio M_Δ/σ would yield smaller f_{ij} couplings,

hence smaller LL leptonic mass insertions, without affecting the RR mass insertions. We remind the reader that, while the overall size of the f_{ij}'s is controlled by M_Δ/σ, their flavour structure, up to small RGE effects, solely depends on the light neutrino parameters. Hence, even though the predictions for LFV processes span many orders of magnitude due to their strong dependence on the unkown scale parameter M_Δ/σ, the ratios of rates for different flavour channels are predicted with a mild dependence on the high-energy parameters, as long as the corrections controlled by the seesaw couplings dominate. This is precisely the case in our numerical example, in which the dominance of the LL mass insertions allows to estimate the ratios of LFV decays, as in usual type II seesaw models [9]:

$$\frac{BR(\tau\to\mu\gamma)}{BR(\mu\to e\gamma)} \approx \frac{|\delta^{LL}_{\mu\tau}|^2}{|\delta^{LL}_{e\mu}|^2}\,\frac{BR(\tau\to\mu\overline{\nu}_\mu\nu_\tau)}{BR(\mu\to e\overline{\nu}_e\nu_\mu)} = \begin{cases} \mathcal{O}(100) & (\sin^2\theta_{13}=0) \\ \mathcal{O}(1) & (\sin^2\theta_{13}=0.05) \end{cases} \tag{32}$$

$$\frac{BR(\tau\to e\gamma)}{BR(\mu\to e\gamma)} \approx \frac{|\delta^{LL}_{e\tau}|^2}{|\delta^{LL}_{e\mu}|^2}\,\frac{BR(\tau\to e\overline{\nu}_e\nu_\tau)}{BR(\mu\to e\overline{\nu}_e\nu_\mu)} = \begin{cases} \mathcal{O}(0.1) & (\sin^2\theta_{13}=0) \\ \mathcal{O}(0.1) & (\sin^2\theta_{13}=0.05) \end{cases}, \tag{33}$$

where we used $BR(\tau\to\mu\overline{\nu}_\mu\nu_\tau)/BR(\mu\to e\overline{\nu}_e\nu_\mu) \simeq BR(\tau\to e\overline{\nu}_e\nu_\tau)/BR(\mu\to e\overline{\nu}_e\nu_\mu) \simeq 0.17$. The present upper bound on $BR(\mu\to e\gamma)$ thus excludes the possibility of observing $\tau\to e\gamma$ in foreseeable experiments, while $\tau\to\mu\gamma$ may be accessible at a super-B factory [20] if θ_{13} is small. These ratios may change if the SO(10) parameters controlling the masses of the heavy states are varied but, as long as Eq. (30) is satisfied, the dependence is only logarithmic, while the branching ratios themselves are roughly proportional to $(M_\Delta/\sigma)^4$.

Let us now turn our attention to CP violation. It is well known that, even if the "flavour-independent" phases carried by the diagonal A-terms and by the mu term (in the convention that the universal gaugino mass parameter is real) are absent, significant contributions to the leptonic and hadronic EDMs may arise from phases carried by flavour non-diagonal soft terms [21, 16, 22]. In the mass insertion approach, this corresponds to multiple insertions of δ's. For down quark and charged lepton EDMs, the dominant "flavour-violating" contribution (coming respectively from gluino and bino diagrams) is proportional to:

$$\mathrm{Im}\left[(\delta^f_{LL})_{i3}\,(\delta^f_{LR})_{33}\,(\delta^f_{RR})_{3i}\right] \approx -\mu\tan\beta\, m_{f_3}\,\mathrm{Im}\left[(\delta^f_{LL})_{i3}\,(\delta^f_{RR})_{3i}\right] \qquad (f=d,e)\ , \tag{34}$$

in which the suppression due to the double flavour change is compensated for by the $\tan\beta\, m_{f_3}$ enhancement. As for the up quark EDM, since $(\delta^u_{RR})_{ij}\approx 0$ its flavour-violating contributions are suppressed by small Yukawa couplings or CKM angles, making it much smaller than the down quark EDM. Together with Eqs. (28) and (29), this implies a correlation between the electron and neutron EDMs, which depend on the same combination of phases and flavour couplings:

$$\mathrm{Im}\left[\mathrm{e}^{i(\Phi^d_3-\Phi^d_1)}\,C^f_{13}\,C^{\lambda_u}_{13}\right]. \tag{35}$$

Using formulae available in the literature (see e.g. Ref. [23]), one can estimate the size of the charged lepton and neutron EDMs for the choice of parameters made in Table 1. With the superpartner masses given above and an average slepton (down squark) mass $\overline{m}_{\tilde{e}} = 1.56\,\mathrm{TeV}$ ($\overline{m}_{\tilde{d}} = 1.8\,\mathrm{TeV}$), one obtains, for $\tan\beta=10$:

$$d_{e_i} \simeq \left(-1.3\times10^{-24}\,e\,\mathrm{cm}\right)\mathrm{Im}\left[(\delta^e_{LL})_{i3}\,(\delta^e_{RR})_{3i}\right], \tag{36}$$

$$d_n \simeq (1\pm0.5)\left(-2.9\times10^{-22}\,e\,\mathrm{cm}\right)\mathrm{Im}\left[(\delta^d_{LL})_{13}\,(\delta^d_{RR})_{31}\right]. \tag{37}$$

For the neutron EDM, we used the formula $d_n = (1\pm 0.5)\,[1.4(d_d - 0.25 d_u) + 1.1 e(d^c_d + 0.5 d^c_u)]$ [24], in which we only kept the dominant gluino contributions to down quark electric and chromo-electric dipole moments, while we neglected the up quark dipole moments. The values of the δ's given in Table 1, together with $|(\delta^d_{LL})_{13}\,(\delta^d_{RR})_{31}| = 2.2\times 10^{-7}$ (for $\theta_{13}=0$) and 3.5×10^{-6} (for $\sin^2\theta_{13} = 0.05$), lead to $d_e \lesssim (7\times 10^{-33} - 10^{-31})\, e\,\mathrm{cm}$, $d_\mu \lesssim 3\times 10^{-30}\, e\,\mathrm{cm}$ and $d_n \lesssim (1\pm 0.5)\,(6\times 10^{-29} - 10^{-27})\, e\,\mathrm{cm}$, where the upper ranges correspond to varying θ_{13} from 0 to its experimental upper limit. These bounds are well below the present experimental upper bounds (respectively $1.6\times 10^{-27}\, e\,\mathrm{cm}$ [25], $1.9\times 10^{-19}\, e\,\mathrm{cm}$ [26] and $2.9\times 10^{-26}\, e\,\mathrm{cm}$ [27]), while some tension with the constraints from LFV processes is already present for the same values of the δ's. Therefore, taking into account the FCNC constraints, we do not expect significant flavour-violating contributions to EDMs in the scenario studied in this paper[4].

A more promising CP-violating observable is the indirect CP violation parameter in the kaon sector, ε_K. A recent evaluation of the Standard Model prediction for ε_K suggests that its measured value still allows for a significant supersymmetric contribution [28]. In the mass insertion language, the relevant contributions are $\mathrm{Im}\,[(\delta^d_{MN})_{12}(\delta^d_{PQ})_{12}]$, with $(M,N),(P,Q)\in \{(L,L),(R,R)\}$ or $\{(L,R),(R,L)\}$. Since the LR mass insertions are suppressed by the down and strange quark masses and $|(\delta^d_{LL})_{12}|$ is suppressed by $|V_{td}V_{ts}|$, the leading contributions to ε_K (for $f_{33}\gtrsim 0.01-0.03$) are proportional to $\mathrm{Im}\,[(\delta^d_{LL})_{12}(\delta^d_{RR})_{12}] \propto R^{-1}\,\mathrm{Im}\,[\,\mathrm{e}^{i(\Phi^d_1-\Phi^d_2)}\,(C^f_{12})^*\,C^{\lambda_u}_{12}]$ and to $\mathrm{Im}\,[(\delta^d_{RR})^2_{12}] \propto \mathrm{Im}\,[\,\mathrm{e}^{2i(\Phi^d_1-\Phi^d_2)}\,(C^{f*}_{12})^2]$, which depend both on CKM/PMNS phases and on the unknown extra SO(10) phases. As discussed at the end of Section 5, this might account for a significant part of the measured value of ε_K.

3.3 Model dependence related to higher-dimensional operators

Throughout the above discussion, we assumed that Eqs. (7) and (8) hold at the GUT scale, yielding the following relations between the masses of the heavy $(L_i,\bar L_i)$ and $(D^c_i,\bar D^c_i)$ fields and of the SM quarks and leptons:

$$M_L \;=\; \frac{V_1}{\sin\theta_H v_d}\, M_e\;, \qquad M_{D^c} \;=\; \frac{V_1}{\sin\theta_H v_d}\, M^T_d\;, \tag{38}$$

as well as $M_d = M^T_e$. The last relation is known to be in gross contradiction with the measured masses of the first two generation fermions, but since the corresponding Yukawa couplings are small, this can easily be cured by higher-dimensional operators involving fields with SU(5)-breaking vevs. These operators also affect, in a model-dependent way, the relations (38) between the masses of the heavy and light matter fields. Moreover, they generally introduce a mismatch between the SO(10) bases in which M_d and M_e are diagonal, as well as between the SO(10) bases in which M_d and M_{D^c} (M_e and M_L) are diagonal. All these effects have an impact on the running of the MSSM soft terms and on the resulting flavour violation. However we argue below that, under reasonable assumptions, their impact on physical observables should remain small.

The first effect, i.e. the fact that the masses of the $(L_i,\bar L_i)$ and $(D^c_i,\bar D^c_i)$ fields are no longer determined by the light fermion masses, introduces only a mild model dependence. Indeed, the heavy masses M_{L_i} and $M_{D^c_i}$ enter Eqs. (11) and (12) [or equivalently Eqs. (26) and (27)] only logarithmically. On top of that, in the case that $M_{24} > M_{L_2,D^c_2}$, m^2_l and $m^2_{d^c}$ only depend on

[4]Of course, "flavour-independent" phases carried by diagonal A-terms or by the mu term may still induce large EDMs.

M_{L_3} and $M_{D_3^c}$, which are presumably little affected by the higher-dimensional operators, while in the case that $M_{24} > M_{L_1, D_1^c}$ some model dependence arises from the contributions of these operators to M_{L_2} and $M_{D_2^c}$. As for $m^2_{e^c}$, it is dominated by M_{L_3}-dependent contributions.

The second effect, i.e. the mismatch between the mass eigenstate basis of the heavy and light quark (lepton) fields, introduces new flavour parameters in the model in the form of unitary matrices connecting a given SO(10) basis to another. As a result, the couplings of the components of the $\mathbf{10}_i$ (namely the l_i, d_i^c, $\bar{L}_i$ and $\bar{D}_i^c$ fields) to the components of the $\mathbf{54}$ are no longer equal at the GUT scale, when expressed in terms of mass eigenstates. For instance, f_Δ and f_Z (see Appendix D for the definition of these couplings) now differ by a (model-dependent) unitary matrix. It follows that the radiative corrections to the MSSM soft terms do not only depend on low-energy flavour parameters, but are also sensitive to these high-energy flavour parameters. However, since the unitary matrices originate from the higher-dimensional operators correcting the masses of the first two generation down quark and charged lepton, it is natural to expect that they are characterized by small mixing angles, thus only mildly affecting the flavour structure of the f_{ij} couplings, which on the contrary is characterized by large θ_{12} and θ_{23} angles. We shall therefore neglect this model dependence in the following, and simply input the $SU(5)$ relations (38) in our numerical study[5].

For completeness, we list below the D=5 operators which can correct the mass relation $M_d = M_e^T$, assuming the field content and pattern of vevs of Appendix A:

$$a_{ij}^{(1,2)} \frac{(\mathbf{16}_i\mathbf{16})_{\mathbf{10}}(\mathbf{10}_j\mathbf{45}_{1,2})_{\mathbf{10}}}{\Lambda} , \quad b_{ij}^{(1,2)} \frac{(\mathbf{16}_i\mathbf{45}_{1,2})_{\mathbf{16}}(\mathbf{10}_j\mathbf{16})_{\overline{\mathbf{16}}}}{\Lambda} , \quad c_{ij} \frac{(\mathbf{16}_i\mathbf{16})_{\mathbf{10}}(\mathbf{10}_j\mathbf{54}')_{\mathbf{10}}}{\Lambda} , \tag{39}$$

where the subscripts specify the contraction of SO(10) indices and Λ is the cutoff. In Eq. (39), the non-vanishing vevs of $\mathbf{45}_1$ and $\mathbf{45}_2$ are along the T_{3R} and $B-L$ directions, respectively, and $\mathbf{54}'$ has a vev in the Pati-Salam singlet direction. The operator $b_{ij}^{(2)}$, for instance, yields the following corrections to the mass matrices of the heavy and light matter fields:

$$\begin{aligned}
(M_e)_{ij} &= \sin\theta_H v_d \left(h_{ij} - 3c\, b_{ij}^{(2)}\right) , \qquad & (M_L)_{ij} &= V_1 \left(h_{ij} + 3c\, b_{ij}^{(2)}\right) , \\
(M_d^T)_{ij} &= \sin\theta_H v_d \left(h_{ij} - c\, b_{ij}^{(2)}\right) , \qquad & (M_{D^c})_{ij} &= V_1 \left(h_{ij} + c\, b_{ij}^{(2)}\right) ,
\end{aligned} \tag{40}$$

where $c = 2\sqrt{2/3}\, V_{B-L}/\Lambda$. Even in the simple case where a single operator ($b^{(2)}$ in this example) is present, the experimental values of the down quark and charged lepton masses are not sufficient to fully determine the couplings h_{ij} and $b_{ij}^{(2)}$. Hence, the heavy field masses and the mixing patterns in the four mass matrices are model dependent. As argued above, however, this is unlikely to affect the radiative corrections to the MSSM soft terms in a sizable way if the couplings h_{ij} and $b_{ij}^{(2)}$ have a hierarchical flavour structure.

4 Model-building aspects

Before presenting our numerical results for flavour violation in the next section, let us briefly present the main ingredients necessary to promote the SO(10) scenario studied in this paper to a realistic model. This includes the dynamics of gauge symmetry breaking, a viable doublet-triplet splitting mechanism and the generation of the intermediate scales associated with the

[5]We note in passing that there is enough freedom in the higher-dimensional couplings (see the example below) to account for the measured down quark and charged lepton masses without introducing new flavour parameters in the model, i.e. with M_d, M_e, M_{D^c} and M_L all diagonal in the same SO(10) basis.

components of the **54** multiplet consistently with gauge coupling unification. More details can be found in Appendices A, B and C.

4.1 Gauge symmetry breaking and doublet-triplet splitting

In order to break the SO(10) gauge group down to $SU(3)_C \times SU(2)_L \times U(1)_Y$, we introduce two adjoint Higgs representations $\mathbf{45}_1$ and $\mathbf{45}_2$ with non-vanishing vevs in the T_{3R} and $B-L$ directions, respectively, and a **54**' which breaks SO(10) down to its Pati-Salam subgroup (we also introduce an SO(10) singlet). The rank of the gauge group is broken by the $(\mathbf{16}, \overline{\mathbf{16}})$ pair. A superpotential that enforces this pattern of vevs is given in Appendix A. In this example, all vevs are proportional to a single mass parameter, M_{16}. Hence, if all superpotential couplings are of order one, SO(10) is broken down to $SU(3)_C \times SU(2)_L \times U(1)_Y$ in one step. Note that the **54**' cannot be identified with the **54** multiplet involved in the seesaw mechanism, since a vev of the latter would induce an unwanted $\bar{\mathbf{5}}_i^{\mathbf{10}} / \bar{\mathbf{5}}_i^{\mathbf{16}}$ mixing (see Section 2), and also for reasons related to the doublet-triplet splitting mechanism.

In order to avoid proton decay at an unacceptable rate, we must split the electroweak doublets from the colour triplets in the **10** and **16** Higgs multiplets. This can be achieved by a generalization of the missing vev mechanism [29] involving an additional $\mathbf{10}'$ Higgs multiplet and an adjoint Higgs representation with a vev aligned along the $B-L$ direction, which is precisely the role of the $\mathbf{45}_2$ (see Appendix B for details). After SO(10) symmetry breaking, all colour triplets acquire GUT-scale masses, and a single pair of Higgs doublets (h_u, h_d) remains massless, the $Y = +1/2$ electroweak doublet in **10** and a combination of the $Y = -1/2$ electroweak doublets in **10** and **16**:

$$h_u \,=\, H_u^{10}\ , \qquad h_d \,=\, \cos\theta_H H_d^{10} + \sin\theta_H H_d^{16}\ , \tag{41}$$

where the Higgs mixing angle is given by $\tan\theta_H = \bar{\eta}\overline{V}_1 / M_{16}$, with $\bar{\eta}$ a superpotential coupling. As discussed in Section 2, $\sin\theta_H \neq 0$ is necessary to give masses to the down quarks and charged leptons, which live in the $\mathbf{10}_i$'s. Furthermore, the leptogenesis scenario of Ref. [7] assumes that h_d contains a non-negligible H_d^{10} component ($\cos\theta_H \neq 0$).

Since the down quark and charged lepton masses are proportional to $\sin\theta_H$ at the renormalizable level, the top/bottom mass hierarchy could in principle be explained by a small Higgs mixing, rather than by a hierarchy in the Yukawa couplings or a large value of $\tan\beta$. This, however, would imply rather heavy vector-like matter fields, since according to Eq. (38) their masses are inversely proportional to $\sin\theta_H$. On the contrary, leptogenesis requires a relatively light $(L_1, \bar{L}_1)$ pair in order to satisfy the condition for a non-vanishing CP asymmetry ($M_\Delta > 2M_{L_1}$). We shall therefore stick to the case $\sin\theta_H \sim 1$.

4.2 Proton decay

Although all colour Higgs triplets acquire GUT-scale masses through the doublet-triplet splitting mechanism, their contribution to the proton decay mode $p \to K^+\bar{\nu}$ can still exceed the experimental limit. To suppress it further, one must impose additional constraints on the combination of doublet-triplet splitting parameters which enters the proton decay rate. More precisely, one must either arrange some cancellation among these parameters, or allow a pair of Higgs doublets to lie at an intermediate scale $M_H \lesssim 10^{14}$ GeV (see Appendix B.2 for details). In this paper, we choose the latter option, which in turn affects the running of the gauge cou-

plings and leads us to split the $(\mathbf{15} \oplus \overline{\mathbf{15}})^{\mathbf{54}}$ and $\mathbf{24}^{\mathbf{54}}$ multiplets in order to restore successful gauge coupling unification, as we discuss below.

4.3 Intermediate scales and gauge coupling unification

The leptogenesis scenario of Ref. [7] requires the $\mathbf{15} \oplus \overline{\mathbf{15}}$ component of the $\mathbf{54}$ to be lighter than its $\mathbf{24}$ component. Actually the full $SU(5)$ multiplets are not needed for leptogenesis: only the $(\Delta, \bar{\Delta})$ pair in $\mathbf{15} \oplus \overline{\mathbf{15}}$, which is responsible for the type II seesaw mechanism, and the S and T fields in $\mathbf{24}$ (or just one of them) are really necessary. The main reason for using complete $SU(5)$ representations in Ref. [7] was gauge coupling unification. However, we have seen that the experimental constraint on the $p \to K^+ \bar{\nu}$ rate can be satisfied by allowing a pair of Higgs doublets to lie at an intermediate scale, which in turn spoils unification. As shown in Appendix C, this can be cured by splitting the masses of the components of $(\mathbf{15} \oplus \overline{\mathbf{15}})^{\mathbf{54}}$ and $\mathbf{24}^{\mathbf{54}}$ in an appropriate manner. This also has the advantage of maintaining perturbativity above the GUT scale, and of avoiding too strong flavour-violating effects in the region of the parameter space relevant for leptogenesis, where the f_{ij} couplings are large and induce important radiative corrections to the MSSM soft terms.

Two simple possibilities emerge from the analysis of gauge coupling unification done in Appendix C (the ratio $M_T / M_\Delta = 10$ is motivated by leptogenesis):

- **model (i):** intermediate $(\Delta, \bar{\Delta})$, $(\Sigma, \bar{\Sigma})$ and T, with

$$M_\Delta = M_\Sigma \ , \qquad M_T = 10 M_\Delta \ , \qquad M_H = 10^{14}\,\text{GeV} \ , \tag{42}$$

- **model (ii):** intermediate $(\Delta, \bar{\Delta})$, $(\Sigma, \bar{\Sigma})$, S, T and O, with (S is actually irrelevant for gauge coupling unification)

$$M_\Sigma^3 / M_\Delta^2 \approx 10^{15}\,\text{GeV} \ , \qquad M_S = M_T = M_O = 10 M_\Delta \ , \qquad M_H = 10^{14}\,\text{GeV} \ , \tag{43}$$

all other components of the $\mathbf{54}$ having GUT-scale masses. The desired splittings can be achieved by appropriate higher-dimensional operators (see Appendix C). In model (i), the unification scale lies almost one order of magnitude lower than in the MSSM, which is at odds with the experimental limit on $p \to \pi^0 e^+$. While this might be cured by 2-loop running and GUT threshold corrections, we prefer to adopt model (ii) for the numerical study of the next section. We note in passing that both models give similar (although quantitatively different) results for flavour violation. For illustration, let us consider model (ii) with $M_\Delta = 10^{12}\,\text{GeV}$, $M_\Sigma = M_S = M_T = M_O = 10^{13}\,\text{GeV}$ and $M_H = 10^{14}\,\text{GeV}$ (the other parameters are chosen to be $V_1 = M_{\text{GUT}}$, $\lambda_H \equiv |\sigma_{\bar{\Delta}}(\mu = M_\Delta)| = 0.045$, $\tan\beta = 10$ and $\tan\theta_H = 1$). The spectrum of heavy states below the GUT scale is shown in Fig. 1(a), and the renormalization group running of the gauge couplings in Fig. 1(b). At the 1-loop level, one finds:

$$M_{\text{GUT}} = 1.2 \times 10^{16}\,\text{GeV} \ , \qquad \alpha_{\text{GUT}} = 1/12.5 \ . \tag{44}$$

The prediction for $\alpha_3(m_Z)$, including supersymmetric thresholds and the 2-loop MSSM running, agrees with the measured value within 1σ. Finally, the Landau pole lies one order of magnitude above M_{GUT}.

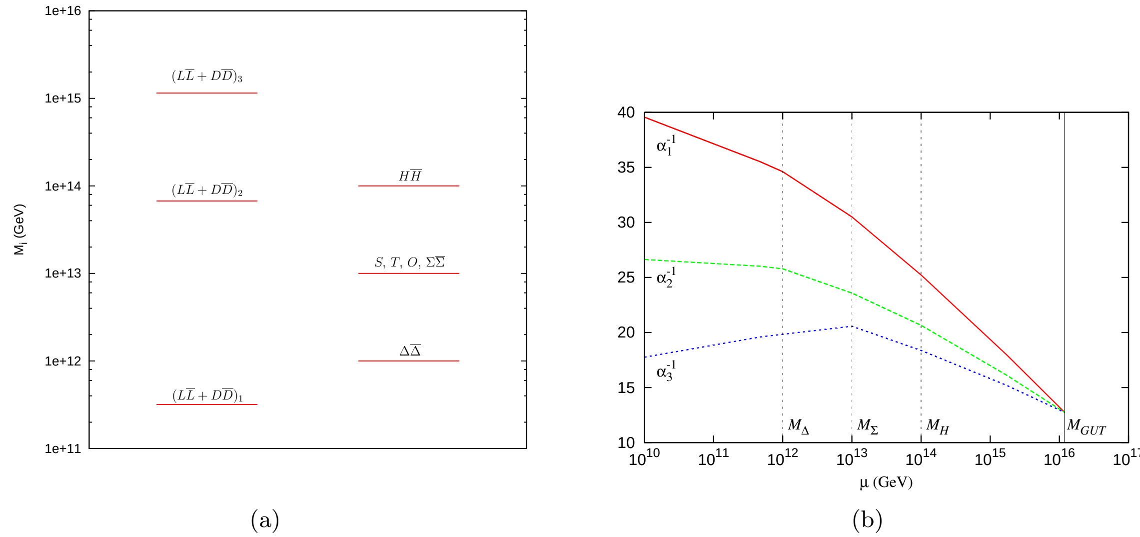


Figure 1: (a) Spectrum of heavy states below the GUT scale in model (ii) for the following choice of parameters: $M_\Delta = 10^{12}$ GeV, $M_\Sigma = M_S = M_T = M_O = 10^{13}$ GeV, $M_H = 10^{14}$ GeV, $V_1 = M_{\rm GUT}$, $\lambda_H \equiv |\sigma_{\bar{\Delta}}(\mu = M_\Delta)| = 0.045$, $\tan\beta = 10$ and $\tan\theta_H = 1$. (b) Renormalization group running of $\alpha_i^{-1}(\mu)$ $(i = 1, 2, 3)$ between $\mu = 10^{10}$ GeV and $\mu = M_{\rm GUT}$.

5 Numerical results

In this section, we perform a numerical analysis of flavour violation in the SO(10) model defined above. Since the flavour-violating effects we want to study arise from radidative corrections to the sfermion soft terms, we numerically solve the full 1-loop RGEs from the unification scale $M_{\rm GUT}$ down to the low-energy scale $M_{\rm EWSB} \equiv \sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}}$, taking into account the presence of additional states and interactions below the GUT scale, which modify the running of the MSSM soft terms as outlined in Section 3. A subset of these RGEs is given in Appendix D, where the definitions of the various superpotential couplings and soft terms can also be found.

In order to set the initial conditions at the GUT scale for the superpotential couplings and to compute the masses of the heavy $(\mathbf{5}_i,\ \bar{\mathbf{5}}_i)$ pairs (which depend on the SO(10) couplings h_{ij}), we first evolve the Yukawa matrices and the low-energy neutrino mass operator from low energy up to the seesaw scale M_Δ, where the couplings f_{ij} are computed according to the seesaw formula $(M_\nu)_{ij} = -\lambda_H f_{ij} v_u^2 / 2M_\Delta$, in which $\lambda_H \equiv |\sigma_{\bar{\Delta}}(\mu = M_\Delta)|$. Then we evolve the f_{ij} together with the other Yukawa couplings up to the GUT scale. This procedure is iterated until convergence of the heavy $(\mathbf{5}_i,\ \bar{\mathbf{5}}_i)$ masses M_{5_i} is reached. The other mass scales are taken as inputs and varied in order to study their impact on leptogenesis and on the flavour-violating observables listed in Table 2. Moreover, to satisfy the proton decay constraint consistently with gauge coupling unification, we split the components of the **54** as indicated in Eq. (43). Thus, specifying the triplet mass M_Δ is enough to fix the masses of all **54** components. The value of M_H has a weaker impact on gauge coupling unification, and we keep it to be 10^{14} GeV. The other inputs in the procedure, apart from the SM and supersymmetric parameters, are the seesaw coupling λ_H, V_1 and $\tan\theta_H$, which are needed to compute the heavy $(\mathbf{5}_i,\ \bar{\mathbf{5}}_i)$ masses.

In order to single out the flavour-violating effects arising from radiative corrections, we assume universal boundary conditions for the soft supersymmetry breaking terms at the GUT

Observable	Bound	Ref.
$\mathrm{BR}(\mu \to e\gamma)$	$< 1.2 \times 10^{-11}$	[30]
$\mathrm{BR}(\mu \to eee)$	$< 1.0 \times 10^{-12}$	[31]
$\mathrm{CR}(\mu \to e \text{ in Ti})$	$< 4.3 \times 10^{-12}$	[32]
$\mathrm{BR}(\tau \to \mu\gamma)$	$< 4.4 \times 10^{-8}$	[33]
$\mathrm{BR}(\tau \to e\gamma)$	$< 3.3 \times 10^{-8}$	[33]
ε_K	$(2.229 \pm 0.012) \times 10^{-3}$	[34]
Δm_K	$(3.483 \pm 0.006) \times 10^{-12}$ MeV	[34]
Δm_{B_s}	(17.77 ± 0.12) ps^{-1}	[34]
$\mathrm{BR}(b \to s\gamma)$	$2.77 \times 10^{-4} - 4.33 \times 10^{-4}$ (3σ)	[35]
$\mathrm{BR}(B_s \to \mu\mu)$	$< 4.7 \times 10^{-8}$	[36]
Δm_B	(0.507 ± 0.005) ps^{-1}	[34]
Δm_D	$(0.0237^{+0.0066}_{-0.0071})$ ps^{-1}	[34]

Table 2: Flavour-violating observables studied in the numerical analysis and their experimental values or upper bounds.

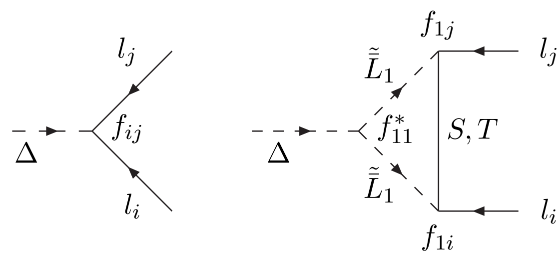


Figure 2: Feynman diagrams responsible (together with the CP-conjugated diagrams) for the CP asymmetry in the decays of the scalar triplet into Standard Model lepton doublets.

scale, with common gaugino mass parameter $M_{1/2}$, scalar soft mass m_0 and A-term a_0. For definiteness, we set $a_0 = 0$ in the following. In a later stage, we shall also comment about the effect of relaxing the equality of the soft mass parameters for different SO(10) multiplets, still assuming flavour-blind supersymmetry breaking. After having performed the running from the GUT scale to low energy, we check that electroweak symmetry breaking does take place and that no tachyonic states are present in the spectrum. We then compute the masses of all Higgs bosons and superpartners and impose the mass limits coming from direct searches at LEP and at the Tevatron. For the rates of LFV processes, we use the expressions of Ref. [37]. The supersymmetric contributions to $\mathrm{BR}(B^0_{d,s} \to \mu^+\mu^-)$ are estimated using the formulae of Ref. [38], while $\mathrm{BR}(b \to s\,\gamma)$ is computed with the help of the routine `SusyBSG` [39]. The supersymmetric contributions to the meson mass splittings Δm_K, Δm_D, Δm_B, Δm_{B_s} and to the indirect CP violation parameter ε_K are computed in the mass insertion approximation, using the formulae of Ref. [40]. We recall in Table 2 the experimental values and upper limits for these observables.

As observed in Section 3.2 and illustrated in Fig. 4, the predictions of the model for LFV processes span many orders of magnitude, due to their strong dependence on the ratio M_Δ/λ_H. In order to obtain definite predictions, we shall restrict the seesaw parameter space to the region favoured by the leptogenesis scenario of Ref. [7], which is built in

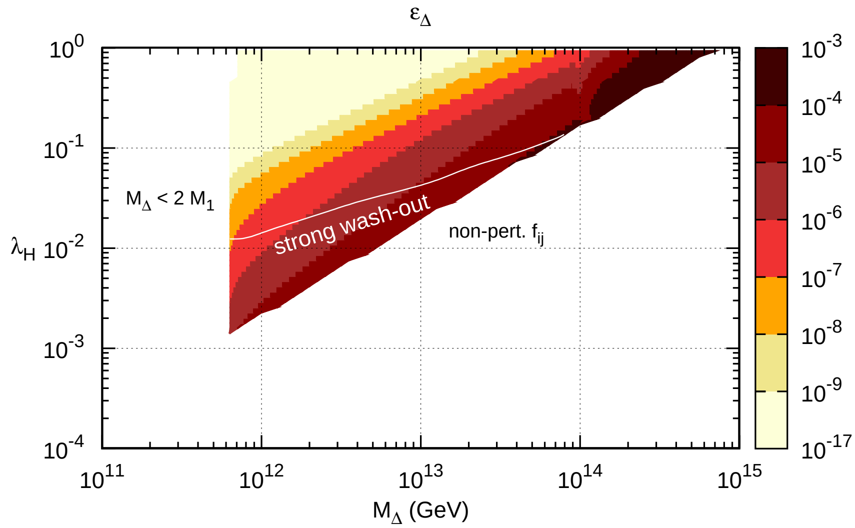


Figure 3: Dependence of the CP asymmetry ε_Δ on M_Δ and λ_H for $\tan\beta = 10$, $V_1 = M_{\rm GUT}$ and maximal Higgs mixing ($\tan\theta_H = 1$). The unmeasured neutrino parameters are chosen to be $m_1 = 0.005\,\text{eV}$, $\sin^2\theta_{13} = 0.05$, $\rho = \pi/4$, $\sigma = \pi/2$ and $\delta = 0$. The area below the white line does not satisfy the conditions for a large efficiency.

the SO(10) model considered here. The source of the lepton asymmetry is the CP asymmetry in triplet decays, which arises from the interference between the tree-level and one-loop diagrams shown in Fig. 2 and is given by (in the limit $M_{L_1} \ll M_\Delta < M_{L_1} + M_{L_2}$): $\varepsilon_\Delta \simeq (0.1/10\pi)\,\text{Im}[f_{11}(f^* f f^*)_{11}] \,/\, \left[\text{Tr}(f f^*) + |f_{11}|^2 + |\lambda_H|^2(1+\cos^4\theta_H)\right]$, where $M_S = M_T = 10 M_\Delta$ has been used. The presence of heavy lepton fields with hierarchical masses is crucial for generating a non-vanishing CP asymmetry. In particular, the condition $M_\Delta > 2 M_{L_1}$ must be satisfied for the loop integral to have an imaginary part.

Let us identify the region of the (M_Δ, λ_H) parameter space in which successful leptogenesis is possible. One can distinguish between two regimes [7]: the first one, characterized by a large value of the CP asymmetry in triplet decays and by a strong washout, requires order one values of the f_{ij} couplings. This however is in conflict with the experimental upper bounds on lepton flavour violation, unless the supersymmetric spectrum is very heavy. In the second regime, the CP asymmetry is smaller, but the efficiency factor accounting for the dilution of the generated lepton asymmetry by washout processes can be of order one. Given that the observed baryon asymmetry is reproduced for $\eta\varepsilon_\Delta \simeq 10^{-8}$, where η is the efficiency factor, successful leptogenesis is possible for $\varepsilon_\Delta \gtrsim 2\times 10^{-8}$. In Fig. 3, we plot ε_Δ as a function of the seesaw parameters M_Δ and λ_H. The light neutrino parameters are chosen as in Ref. [7], with the best fit values for the measured oscillation parameters taken from Ref. [15] and $m_1 = 0.005\,\text{eV}$, $\sin^2\theta_{13} = 0.05$, $\rho = \pi/4$, $\sigma = \pi/2$ and $\delta = 0$. In the region $M_\Delta \lesssim 6\times 10^{11}\,\text{GeV}$, the condition for a non-vanishing CP asymmetry, $M_\Delta > 2M_{L_1}$, is not satisfied, while in the small λ_H region some of the f_{ij} couplings become non-perturbative below the GUT scale, making the computation of ε_Δ non reliable. The white line separates the weak and strong washout

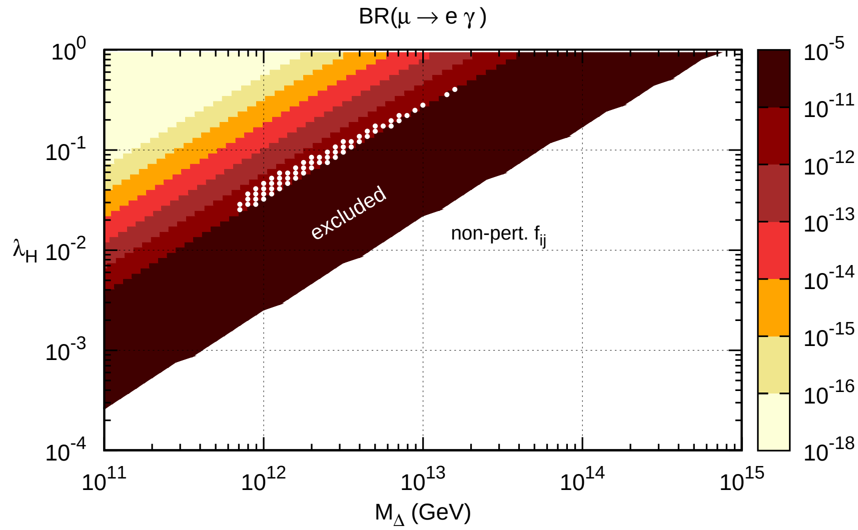


Figure 4: Dependence of BR($\mu \to e\gamma$) on M_Δ and λ_H, for the following choice of mSUGRA parameters: $m_0 = M_{1/2} = 700$ GeV, $a_0 = 0$, $\tan\beta = 10$ and $\mu > 0$. The other parameters are chosen as in Fig. 3. The white dots indicate the area where both BR($\mu \to e\gamma$) $< 1.2 \times 10^{-11}$ and $\varepsilon_\Delta > 2 \times 10^{-8}$.

regimes: below this line, the conditions for a large efficiency, $\Gamma(\Delta \to \bar{\tilde{L}}_1 \bar{\tilde{L}}_1) < H(M_\Delta)$ and $\Gamma(\Delta \to H_u H_u) > H(M_\Delta)$, are not satisfied (see Ref. [7] for details). As shown by Fig. 3, ε_Δ can reach sizable values in the large efficiency region.

Let us now see how the LFV decay $\mu \to e\gamma$ constrains the parameter space of Fig. 3. This is shown in Fig. 4 for a point of the MSSM (mSUGRA) parameter space giving a rather light superpartner spectrum: $m_0 = M_{1/2} = 700$ GeV, $a_0 = 0$, $\tan\beta = 10$ and $\mu > 0$. The brown (dark) area is excluded by the experimental upper limit BR($\mu \to e\gamma$) $< 1.2 \times 10^{-11}$. Comparing Fig. 4 with Fig. 3, one observes a tension between the requirement of successful leptogenesis and the experimental constraint on $\mu \to e\gamma$, due to the fact that both ε_Δ and BR($\mu \to e\gamma$) grow with the ratio M_Δ/λ_H (to which the f_{ij} couplings are proportional). Indeed, the region of the (M_Δ, λ_H) parameter space consistent with both BR($\mu \to e\gamma$) $< 1.2 \times 10^{-11}$ and $\varepsilon_\Delta > 2 \times 10^{-8}$, marked with white dots in Fig. 4, is rather small for the chosen MSSM parameters[6]. An heavier superpartner spectrum and/or a smaller value of $\tan\beta$ would increase the size of this region by relaxing the $\mu \to e\gamma$ constraint, without affecting leptogenesis. A smaller value of θ_{13} would also relax the $\mu \to e\gamma$ constraint, but it would simultaneously reduce ε_Δ. Nevertheless, the requirement of successful leptogenesis (together with the non-observation of $\mu \to e\gamma$) considerably restricts the seesaw parameter space, thus allowing us to make testable predictions for flavour-violating observables.

In the following, we present our results for a set of seesaw parameters belonging to the region

[6]In passing, this shows that the supersymmetric version of the leptogenesis scenario proposed in Ref. [7] can be excluded on the basis of low-energy flavour physics measurements, if the supersymmetric partners are accessible at the LHC (and barring cancellation with other sources of flavour violation in the soft terms).

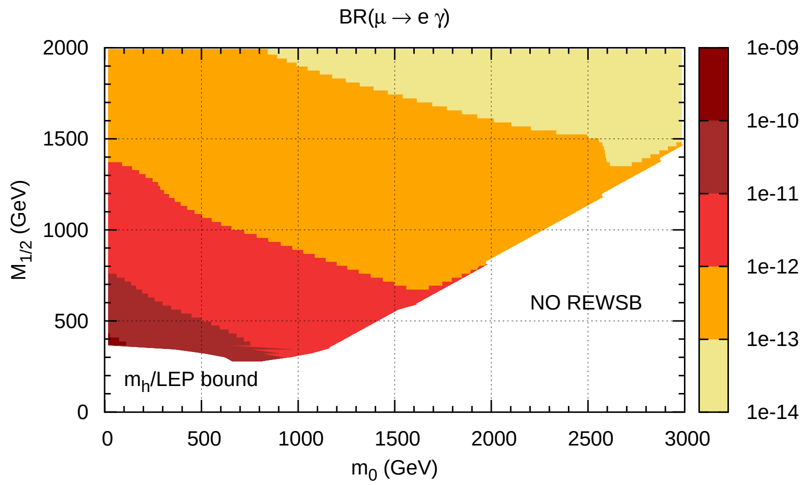
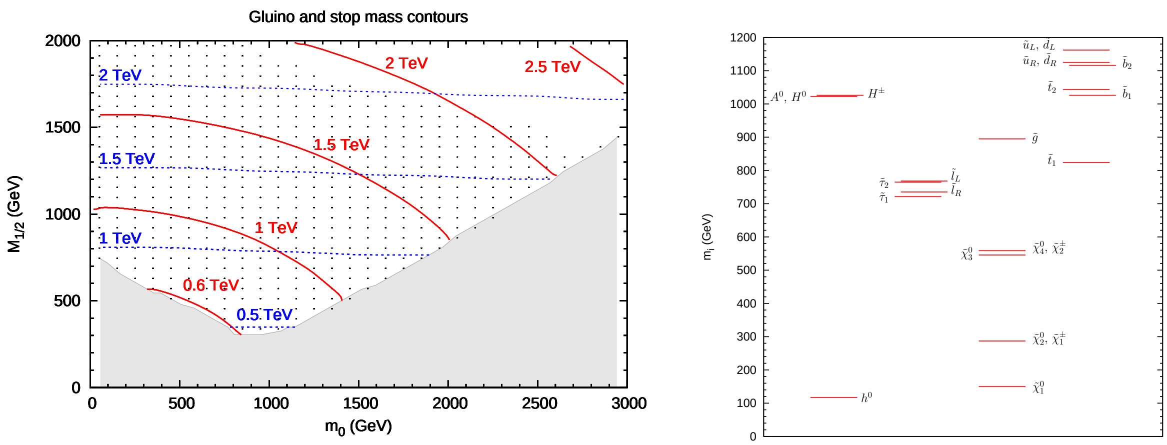


Figure 5: Predictions for BR($\mu \to e\gamma$) in the (m_0, $M_{1/2}$) plane, assuming $a_0 = 0$, $\tan\beta = 10$ and $\mu > 0$. The parameters of the SO(10) model are: $M_\Delta = 10^{12}$ GeV, $\lambda_H = 0.045$, $V_1 = M_{\rm GUT}$ and $\tan\theta_H = 1$. The neutrino parameters are chosen as in Fig. 3.

where successful leptogenesis is possible, namely we take $M_\Delta = 10^{12}$ GeV and $\lambda_H = 0.045$, together with $V_1 = M_{\rm GUT}$ and $\tan\theta_H = 1$ (which gives $\varepsilon_\Delta \simeq 2.5 \times 10^{-8}$). The corresponding spectrum of heavy states is shown in Fig. 1(a). As for the neutrino parameters, we choose them as in Fig. 3. We are now ready to present the predictions of the model for various flavour-violating observables as a function of the MSSM parameters. In Fig. 5, the contours of BR($\mu \to e\gamma$) are plotted in the (m_0, $M_{1/2}$) plane for $a_0 = 0$, $\tan\beta = 10$ and $\mu > 0$ (the contours corresponding to different values of $\tan\beta$ can be easily deduced from Fig. 5 by noting that BR($\mu \to e\gamma$) approximately scales as $\tan^2\beta$). The parameter space is bounded by the LEP limits on superpartner and Higgs boson masses for low values of $M_{1/2}$, and by the absence of radiative electroweak symmetry breaking for large values of m_0 and moderate $M_{1/2}$. BR($b \to s\gamma$) gives a constraint similar to the Higgs mass bound, while the other hadronic observables of Table 2 do not significantly restrict the parameter space. This implies that the flavour physics signatures of the model are expected to show up in the lepton sector rather than in the hadronic sector (with the possible exception of ε_K discussed at the end of this section). One can see from Fig. 5 that the on-going experiment MEG [41], which aims at a sensitivity of $\mathcal{O}(10^{-13})$ on BR($\mu \to e\gamma$), will probe the model over a large portion of the MSSM parameter space. Interestingly, this region approximately corresponds to the one that is accessible at the LHC. This can be seen in Fig. 6(a), where the contours of the gluino and of the lightest stop masses are plotted in the $(m_0, M_{1/2})$ plane, and the area that will be probed by MEG is marked with dots. For completeness, we show in Fig. 6(b) the full supersymmetric spectrum corresponding to the mSUGRA point of Fig. 4, i.e. $m_0 = M_{1/2} = 700\,$GeV, $a_0 = 0$, $\tan\beta = 10$ and $\mu > 0$. Note that this spectrum is characterized by rather heavy sfermions compared to neutralinos and charginos. This is due to the relatively large value of the unified gauge coupling, which enhances the gauge contributions to the running of sfermion masses at high energy. As a result, the lightest neutralino is found to be the LSP over the whole (m_0,



(a) (b)

Figure 6: (a) Contours of the gluino mass (blue dashed lines) and of the lightest stop mass (red solid lines) in the $(m_0, M_{1/2})$ plane, for the same choice of parameters as in Fig. 5. The grey region is excluded by the constraints discussed in the text, including the experimental upper limit on BR$(\mu \to e\gamma)$. The dotted area will be probed by MEG. (b) Supersymmetric spectrum corresponding to the mSUGRA point $m_0 = M_{1/2} = 700\,\text{GeV}$, $a_0 = 0$, $\tan\beta = 10$ and $\mu > 0$.

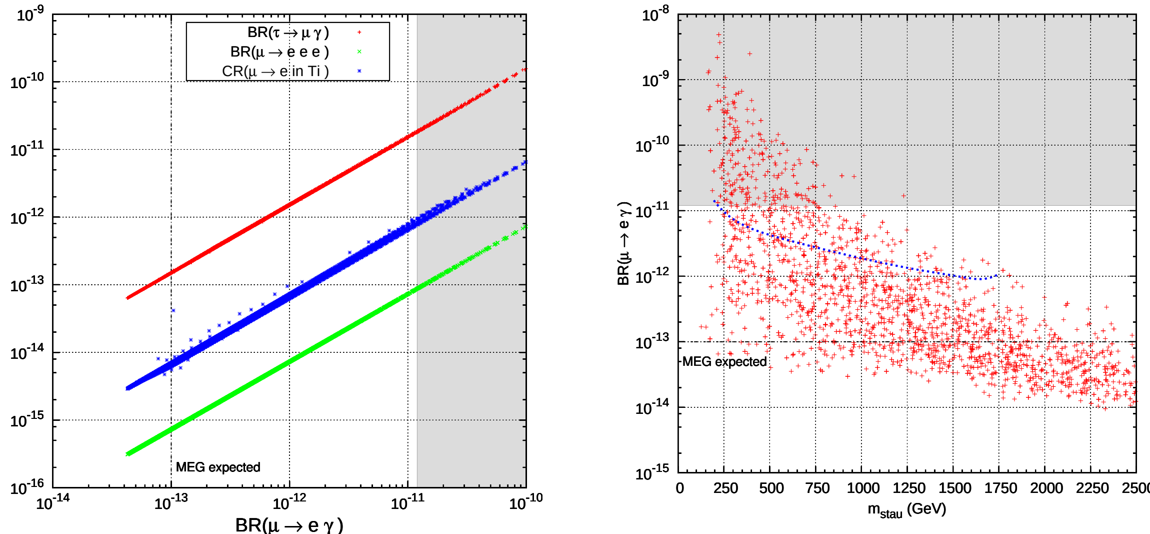
$M_{1/2}$) parameter space. For the spectrum showed in Fig. 6(b), the sleptons are heavier than all neutralinos and charginos and cannot be produced in cascade decays of squarks at the LHC. However, this possibility is recovered for $M_{1/2} \gg m_0$.

Let us now present the predictions of the model for other LFV processes. In Fig. 7(a), BR$(\tau \to \mu\gamma)$, BR$(\mu \to eee)$ and the $\mu - e$ conversion rate in the Titanium nucleus are plotted against BR$(\mu \to e\gamma)$ for the same parameter choice as in Fig. 5. BR$(\tau \to e\gamma)$ is not shown since, as discussed in Section 3.2, it is generally smaller than BR$(\tau \to \mu\gamma)$. Given the experimental upper limits shown in Table 2, BR$(\mu \to e\gamma)$ is at present the most constraining LFV observable. Since BR$(\tau \to \mu\gamma)$/BR$(\mu \to e\gamma) = \mathcal{O}(1)$, in agreement with Eq. (32) and with the correlation observed in type II seesaw models for large θ_{13} [9], $\tau \to \mu\gamma$ is out of reach of super B factories, which are expected to achieve a sensivity of 10^{-9} on its branching ratio [20]. $\mu - e$ conversion looks more promising, given that proposed experiments at Fermilab [42] and at J-PARK [43] aim at respective sensitivities of 10^{-16} and 10^{-18} on CR$(\mu \to e$ in Ti). If approved, these experiments would test the model well beyond the MEG reach.

In order to estimate the impact of non-universal (but flavour-blind) boundary conditions on the predictions for LFV observables, we performed a random scan of the soft scalar masses for different SO(10) multiplets between 0 and 3 TeV, assuming a fixed value $M_{1/2} = 700\,\text{GeV}$ of the common gaugino mass parameter. The results for BR$(\mu \to e\gamma)$ are shown in Fig. 7(b), where the blue dotted line[7] corresponds to the universal case $m_{16} = m_{10} = m_{16_H} = m_{10_H} = m_{54} \equiv m_0$. Relaxing the universality of soft scalar masses can enhance or suppress BR$(\mu \to e\gamma)$ by up to 2 orders of magnitude, but most of the points still remain within the reach of MEG (unless the lightest slepton is very heavy).

As mentioned earlier, LFV observables provide much stronger constraints on the model

[7]In the universal case, radiative electroweak symmetry breaking does not take place for large m_0 values, which explains why the blue dotted line stops at $m_{\tilde{\tau}_1} \approx 1750\,\text{GeV}$.



(a) (b)

Figure 7: (a) BR($\tau \to \mu\gamma$), BR($\mu \to eee$) and CR($\mu \to e$ in Ti) versus BR($\mu \to e\gamma$) for the same choice of parameters as in Fig. 5, scanning over m_0 and $M_{1/2}$ in the ranges $0 < m_0 < 3\,\text{TeV}$ and $0 < M_{1/2} < 2\,\text{TeV}$. The grey area is excluded by the experimental upper limit on BR($\mu \to e\gamma$). (b) BR($\mu \to e\gamma$) as a function of the lightest slepton mass for non-universal soft scalar masses m_{16}, m_{10}, m_{16_H}, m_{10_H} and m_{54} in the $[0, 3\,\text{TeV}]$ range, and $M_{1/2} = 700\,\text{GeV}$. The blue dotted line corresponds to the universal case $m_{16} = m_{10} = m_{16_H} = m_{10_H} = m_{54} \equiv m_0$.

studied in this paper than hadronic observables. A possible exception is represented by the indirect CP violation parameter in the kaon sector, ε_K. It is well known that ε_K is very sensitive to new sources of flavour and CP violation in the 1-2 down squark sector. In the present model, radiative corrections generate large off-diagonal entries both in the LL slepton mass matrix (which leads to large LFV rates) and in the RR down squark mass matrix. Moreover, order one phases in the PMNS matrix, hence in the f_{ij} couplings, are needed to account for the baryon asymmetry of the universe. Unfortunately, unknown extra SO(10) phases spoil the link between CP violation in the neutrino sector and the RG-induced CP violation in the sfermion mass matrices (see the discussion in Section 3.2). Nevertheless, barring cancellations between contributions carrying different phases, it is reasonable to expect a large imaginary part of $(\delta^d_{RR})_{12}$. In Fig. 8, the contours of the supersymmetric contribution to ε_K are plotted in the $(m_0, M_{1/2})$ plane for the same choice of parameters as in Fig. 5, assuming $\arg\left[(\delta^d_{RR})_{12}\right] = 0.5$. While low values of $M_{1/2}$ would give too large a contribution to ε_K, a rather light superpartner spectrum can account for up to $(10-20)\%$ of its experimental value. According to Ref. [28], this is precisely what is needed in order to reconcile the SM prediction for ε_K with experiment.

6 Conclusions

We have studied flavour violation in a supersymmetric SO(10) implementation of the type II seesaw mechanism, which provides a predictive realization of triplet leptogenesis. In this scenario, the high-energy flavour parameters involved in the computation of the bayon asymmetry of the universe and of flavour-violating observables are determined in terms of the Standard

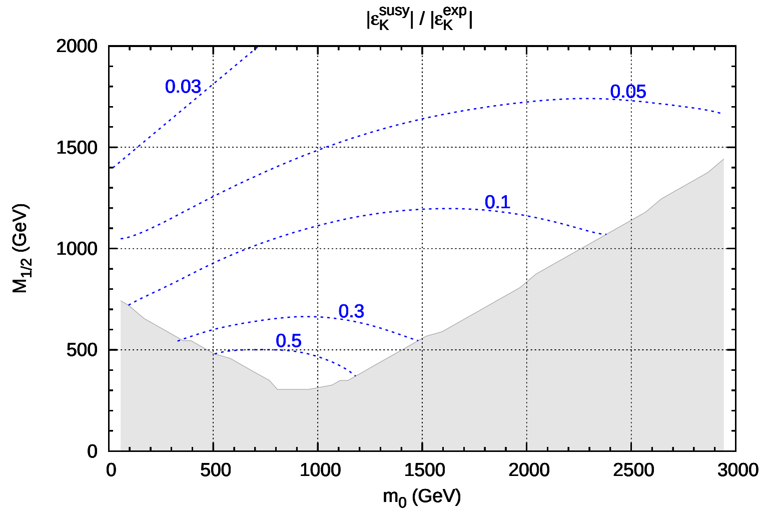


Figure 8: Contours of $|\varepsilon_K^{\rm susy}|/|\varepsilon_K^{\rm exp}|$ in the $(m_0, M_{1/2})$ plane (where $\varepsilon_K^{\rm susy}$ is the supersymmetric contribution to ε_K and $\varepsilon_K^{\rm exp}$ its central experimental value), for the same choice of parameters as in Fig. 5 and assuming arg $\left[(\delta^d_{RR})_{12}\right] = 0.5$.

Model fermion masses and mixing, up to mild model-dependent uncertainties. The overall size of the FCNC effects is then controlled by a few unknown flavour-blind parameters, while the ratios of FCNC rates for different flavour channels, such as $BR(\tau \to \mu\gamma)/BR(\mu \to e\gamma)$, mainly depend on low-energy parameters.

The features of flavour violation in the SO(10) scenario studied in this paper present some interesting differences with the SU(5) implementation of the type II seesaw mechanism [9], because of additional contributions coming from the heavy matter fields crucial for leptogenesis. These give rise, for example, to radiative corrections to the soft mass matrices $m^2_{e^c}$ and m^2_q controlled by the top quark Yukawa coupling. Moreover, the presence of a built-in leptogenesis mechanism provides a criterion for fixing the values of the unknown flavour-blind parameters, thus yielding testable predictions for the rates of flavour-violating processes. Imposing the conditions for successful leptogenesis together with the experimental constraints on FCNCs, we found that the predicted branching ratio for $\mu \to e\gamma$ lies within the sensitivity of the MEG experiment if the superpartner spectrum is accessible at the LHC, while $\tau \to \mu\gamma$ is out of reach of future super B factories. $\mu - e$ conversion on Titanium is also a promising process, with a predicted rate within the reach of proposed experiments at Fermilab and J-PARK even in regions of the parameter space where $BR(\mu \to e\gamma)$ lies below the MEG sensitivity. Hadronic observables only receive small contributions once the experimental bounds on LFV processes are imposed, with the possible exception of ε_K.

We have also studied flavour-violating contributions to leptonic and hadronic EDMs, as well as to the ε_K parameter. The CKM and PMNS phases, together with new CP-violating phases associated with the SO(10) structure, can in principle induce sizable contributions to these observables, even if the soft terms are real at the high scale. The experimental bounds on LFV processes, however, prevent significant contributions to the EDMs, while a sizable contribution to ε_K (which might be necessary to account for its measured value) is still possible.

The predictivity of the scenario studied in this paper relies on the SO(10) relations between

the flavour structures of the SM and heavy matter fields. These relations, however, only hold at the tree level and may be affected by model-dependent corrections from the non-renormalizable operators necessary to account for the measured quark and lepton masses. Nevertheless, the impact of these corrections on our results can be estimated to be mild.

The presence of heavy states below the GUT scale, besides inducing low-energy flavour and CP violation through radiative corrections, also has an impact on the superpartner spectrum. Indeed, their contribution to the beta functions leads to a relatively large value of the unified gauge coupling, which enhances the gauge contribution to the running of sfermion masses. One of the consequences of this is that the lightest neutralino turns out to be the LSP in an unusually large portion of the parameter space.

Finally, we have shown that the SO(10) implementation of the type II seesaw mechanism studied in this paper can be promoted to a consistent model including the dynamics of gauge symmetry breaking, doublet-triplet splitting consistent with the present experimental bounds on proton decay, and gauge coupling unification. The doublet-triplet splitting is achieved through a generalization of the missing vev mechanism. The proton decay rate is suppressed by arranging, without fine-tuning, a pair of Higgs doublets to lie at an intermediate scale. Gauge coupling unification is then restored by an appropriate splitting of the SU(5) components of the **54** containing the type II seesaw triplet, which in turn brings $\alpha_3(m_Z)$ within 1σ from its experimental value, thus improving on the MSSM prediction. In passing, we provided an updated, comprehensive analysis of proton decay from D=5 and D=6 operators.

Acknowledgments

We thank F. Joaquim and A. Rossi for useful comments. The work of MF was supported in part by the Marie-Curie Intra-European Fellowship MEIF-CT-2007-039968. AR acknowledges partial support from the RTN European Program "UniverseNet" (MRTN-CT-2006-035863).

A SO(10) gauge symmetry breaking

The purpose of this appendix is to provide an explicit sector breaking the SO(10) gauge symmetry down to the SM gauge group. It is well known that this breaking can be realized by the most general renormalizable superpotential involving the following Higgs representations: one **45**, one **54** and one $(\mathbf{16}, \overline{\mathbf{16}})$ pair. In this case, both SM singlets in the **45** acquire a nonzero vev. However, we need a **45** multiplet with a vev aligned in the $B-L$ direction to implement the missing vev mechanism for doublet-triplet splitting (see Appendix B). In order to obtain such a vacuum alignment, we must introduce additional fields and consider a non-generic superpotential. A simple realization of this is provided by the following superpotential:

$$W_{\rm SB} \;=\; \frac{1}{2} f_1 \mathbf{54'} \mathbf{45}_1 \mathbf{45}_1 + (\lambda_{12} \mathbf{S} + f_{12} \mathbf{54'}) \mathbf{45}_1 \mathbf{45}_2 + \frac{1}{3} \lambda\, \mathbf{54'} \mathbf{54'} \mathbf{54'} + \overline{\mathbf{16}} (M_{16} + g \mathbf{45}_1) \mathbf{16}\ , \quad \text{(A.1)}$$

where $\mathbf{S}$ is an SO(10) singlet and the contractions of SO(10) indices are understood. As shown below, the $\mathbf{54'}$ acquires a GUT-scale vev and therefore cannot be identified with the **54** multiplet involved in the type II seesaw mechanism.

Altogether, eight SM singlets can acquire a nonzero vev at the GUT scale. We normalize their vevs as follows: $\langle \mathrm{Tr}(\mathbf{54'}^\dagger \mathbf{54'}) \rangle = |V_{PS}|^2$, which breaks SO(10) down to its Pati-Salam subgroup, $\langle \mathrm{Tr}(\mathbf{45}_i^\dagger \mathbf{45}_i) \rangle = |V^{(i)}_{B-L}|^2 + |V^{(i)}_R|^2$ $(i=1,2)$, $\langle \mathbf{16}^\dagger \mathbf{16} \rangle = |V_1|^2$, $\langle \overline{\mathbf{16}}^\dagger \overline{\mathbf{16}} \rangle = |\overline{V}_1|^2$ and $\langle \mathbf{S} \rangle = S$. In order to preserve supersymmetry, these vevs must satisfy F-flatness and D-flatness conditions. The solution to these constraints reads[8]:

$$\begin{aligned}
V^{(1)}_{B-L} &= V^{(2)}_R = 0\ , \qquad & V^{(1)}_R &= \frac{1}{2g} M_{16}\ , \qquad & V^{(2)}_{B-L} &= -\frac{3\sqrt{3} f_1}{10\sqrt{2} f_{12} g} M_{16}\ , \\
V^2_{PS} &= -\frac{3 f_1}{8 \lambda g^2} M^2_{16}\ , \qquad & S &= -\frac{\sqrt{3} f_{12}}{2\sqrt{5} \lambda_{12}} V_{PS}\ , \qquad & \overline{V}_1 V_1 &= \frac{\sqrt{3} f_1}{8\sqrt{5} g^2} V_{PS} M_{16}\ ,
\end{aligned} \quad \text{(A.2)}$$

together with $|V_1| = |\overline{V}_1|$. If the dimensionless couplings are of order one, then all nonzero vevs are of order M_{16}, the unique mass parameter in $W_{\rm SB}$. Therefore SO(10) is broken down to the SM gauge group in one step. The two **45** vevs are aligned along the T_{3R} and $B-L$ directions, respectively; in the rest of the paper we will rename them $V_R \equiv V^{(1)}_R$ and $V_{B-L} \equiv V^{(2)}_{B-L}$.

The SM vacuum defined by Eq. (A.2) provides all uneaten chiral superfields with a GUT-scale mass, except for a pair of charged states with $SU(3)_C \times SU(2)_L \times U(1)_Y$ quantum numbers $(1,1,\pm 1)$, which remains massless[9]. This problem can be cured by adding the following terms to $W_{\rm SB}$:

$$\frac{1}{2}\, \lambda_3\, \mathbf{S}\, \mathbf{45}_3 \mathbf{45}_3 \;+\; \lambda_{123}\, \mathbf{45}_1 \mathbf{45}_2 \mathbf{45}_3\ . \quad \text{(A.3)}$$

This does not modify the vacuum alignment discussed above and does not introduce extra mass parameters either. The F-flatness conditions imply that the $\mathbf{45}_3$ does not acquires a vev. The coupling λ_{123} is sufficient to make the unwanted massless states heavy, while the coupling λ_3 guarantees that all $\mathbf{45}_3$ components are also massive.

[8] Let us mention for completeness that the D-flatness and F-flatness conditions admit another solution characterized by $V^{(2)}_{B-L} = V^{(1)}_R = 0$ and all other vevs nonzero. This solution would also be satisfactory for our purposes, but we stick to the first one for definiteness.

[9] In the alternative vacuum characterized by $V^{(2)}_{B-L} = V^{(1)}_R = 0$, the massless states are $(\overline{3}, 1, -2/3) \oplus (3, 1, 2/3)$.

For completeness, we give the masses of the X and Y gauge bosons, which can mediate proton decay through $D = 6$ operators (see Appendix B.3):

$$M_X^2 \ = \ M_Y^2 \ = \ g_{\rm GUT}^2 \left(\frac{5}{6}\, |V_{PS}|^2 + \frac{1}{3}\, |V_{B-L}|^2 + \frac{1}{2}\, |V_R|^2 \right) \ , \qquad \text{(A.4)}$$

where $g_{\rm GUT}$ is the SO(10) gauge coupling.

We remark that the desired vev alignment is obtained because several couplings allowed by the SO(10) gauge symmetry are absent in Eq. (A.1). In fact, one can add mass terms for $\mathbf{45}_1$ and $\mathbf{54}'$ without upsetting the vev alignment. On the contrary, it is crucial to forbid all $\mathbf{45}_2$ couplings except λ_{12} and f_{12}. We do not try to justify the non-generic form of $W_{\rm SB}$ by global symmetries, which would require a more complicated set of fields and interactions.

Note finally that it is possible to align the vev of one of the two $\mathbf{45}$'s without introducing the SO(10) singlet $\mathbf{S}$. In fact, if one replaces $\lambda_{12}\, \mathbf{S}$ in Eq. (A.1) by a bare mass M_{12}, the F-flatness equations still have a solution with $V_R^{(1)} = 0$ and a second one with $V_{B-L}^{(1)} = 0$, all other vevs being nonzero. This is sufficient to realize the doublet-triplet splitting, but this requires the presence of two mass parameters (M_{16} and M_{12}) in $W_{\rm SB}$.

B Doublet-triplet splitting and proton decay

In this appendix, we provide an explicit doublet-triplet splitting mechanism for the SO(10) scenario studied in this paper, and derive the conditions imposed by the non-observation of proton decay on the heavy spectrum.

B.1 Doublet-triplet splitting

In the SO(10) implementation of the type II seesaw mechanism considered in this paper, up quark masses arise from the superpotential couplings $\mathbf{16}_i \mathbf{16}_j \mathbf{10}$, while down quark and charged lepton masses arise from the couplings $\mathbf{16}_i \mathbf{10}_j \mathbf{16}$. The $Y = +1/2$ MSSM Higgs doublet h_u should therefore reside mainly in the $\mathbf{10}$ in order to account for the large top quark mass, while the $Y = -1/2$ Higgs doublet h_d should contain a significant H_d^{16} component, and we require that it also has a non-vanishing H_d^{10} component[10]. The doublet-triplet splitting mechanism, besides rendering all colour triplets coupling to light matter fields heavy, should therefore leave H_u^{10} and an admixture of H_d^{10} and H_d^{16} massless.

This can be achieved by a generalization of the missing vev mechanism [29] involving an additional $\mathbf{10}'$ Higgs multiplet and an adjoint Higgs field with a non-vanishing vev in the $B-L$ direction (which motivates the choice made for $\mathbf{45}_2$ in Appendix A). The superpotential that accomplishes the desired doublet-triplet splitting reads:

$$W_{\rm DT} \ = \ \frac{1}{2}\, M_{10'}\, \mathbf{10}' \mathbf{10}' + h\, \mathbf{10}' \mathbf{45}_2 \mathbf{10} + \overline{\mathbf{16}} (M_{16} + g\, \mathbf{45}_1) \mathbf{16} + \frac{1}{2}\, \overline{\mathbf{16}}\, \overline{\mathbf{16}}\, (\bar \eta \mathbf{10} + \bar \rho \mathbf{10}') \ , \qquad \text{(B.1)}$$

where the third term also plays a role in SO(10) symmetry breaking (and therefore appears in Eq. (A.1)), while the coupling $\bar \rho\, \overline{\mathbf{16}}\, \overline{\mathbf{16}}\, \mathbf{10}'$ is optional. As in the case of Eq. (A.1), this is not the most general superpotential for the fields involved. After SO(10) symmetry breaking, one

[10] Indeed, the leptogenesis scenario of Ref. [7] assumes that the dominant decay modes of the heavy matter fields preserve $B-L$, which is guaranteed if h_d contains a non-negligible H_d^{10} component.

obtains the following doublet and triplet mass matrices, written in the $(\mathbf{10},\mathbf{10}',\mathbf{16})$ (left) and $(\mathbf{10},\mathbf{10}',\overline{\mathbf{16}})$ (right) bases:

$$M_D \;=\; \begin{pmatrix} 0 & 0 & -\bar{\eta}\overline{V}_1 \\ 0 & M_{10'} & -\bar{\rho}\overline{V}_1 \\ 0 & 0 & M_{16} \end{pmatrix} , \quad M_T \;=\; \begin{pmatrix} 0 & \frac{h}{\sqrt{6}}V_{B-L} & -\bar{\eta}\overline{V}_1 \\ -\frac{h}{\sqrt{6}}V_{B-L} & M_{10'} & -\bar{\rho}\overline{V}_1 \\ 0 & 0 & M_{16}+2gV_R \end{pmatrix} . \tag{B.2}$$

From Eq. (B.2) we can see that all colour triplets and two pairs of Higgs doublets acquire GUT-scale masses, while H_u^{10} and a combination of H_d^{10} and H_d^{16} remain massless:

$$h_u \;=\; H_u^{10} \;, \qquad h_d \;=\; \cos\theta_H H_d^{10} + \sin\theta_H H_d^{16} \;, \tag{B.3}$$

where the Higgs mixing angle θ_H is given by (with no loss of generality, we assume $\bar{\eta}\overline{V}_1$ and M_{16} to be real):

$$\tan\theta_H \;=\; \frac{\bar{\eta}\overline{V}_1}{M_{16}} \;. \tag{B.4}$$

Note that the alignment of the $\mathbf{45}_2$ vev along the $B-L$ direction is crucial for the splitting of the doublet and triplet components.

A few comments are in order about the above doublet-triplet mechanism. First, the SO(10)-breaking sector contains a $\mathbf{54}'$ representation with a vev in the Pati-Salam direction, whose couplings to $\mathbf{10}\,\mathbf{10}$ and $\mathbf{10}\,\mathbf{10}'$ must be forbidden as they would make all Higgs doublets heavy. On the contrary, the coupling $\mathbf{10}\,\mathbf{10}\,\mathbf{54}$ involved in the seesaw mechanism is harmless since $\mathbf{54}$ does not acquire a vev. Second, the mass term $M_{16}\overline{\mathbf{16}}\mathbf{16}$ in Eqs. (A.1) and (B.1) is crucial both for GUT symmetry breaking and for the doublet-triplet splitting: if it were absent, F-flatness would imply either $V_1 = 0$, leaving the SO(10) rank unbroken, or $V_R = 0$, leaving a pair of colour triplets massless.

B.2 Higgs-mediated proton decay

In general, giving GUT-scale masses to the colour Higgs triplets is not enough to suppress the proton decay rate below its experimental upper limit. To achieve this, one must impose additional constraints on the triplet mass matrix. We show below that this can be done in a simple way in the doublet-triplet splitting scenario presented above.

Let us first adapt the standard computation of the Higgs-mediated proton decay rate [44, 45, 46, 47] to our case. Upon integrating out the heavy Higgs triplet superfields, one obtains the following D=5 operators:

$$\frac{1}{2}\,\kappa_{ijkl}\,(Q_iQ_j)(Q_kL_l)\,\Big|_{\theta^2} \;+\; \frac{1}{2}\,\kappa'_{ijkl}\,u^c_iu^c_jd^c_ke^c_l\,\Big|_{\theta^2} \;+\; \text{h.c.}\,, \tag{B.5}$$

where the contraction of gauge indices is understood, and the dimensionful coefficients κ_{jikl} and κ'_{jikl} generated by the superpotential (1) are given by[11]:

$$\kappa_{ijkl} \;=\; -\,y_{ij}h_{kl}\left(M_T^{-1}\right)_{T^{10}\overline{T}^{16}} \;, \qquad \kappa'_{ijkl} \;=\; -\left(y_{il}h_{jk}-y_{jl}h_{ik}\right)\left(M_T^{-1}\right)_{T^{10}\overline{T}^{16}} \;, \tag{B.6}$$

where $\left(M_T^{-1}\right)_{T^{10}\overline{T}^{16}}$ is the $(T^{10},\overline{T}^{16})$ entry of the inverse triplet mass matrix. Since colour invariance implies $\kappa_{iiil} = 0$ and $\kappa'_{iikl} = 0$, the dominant proton decay modes arising from

[11]Note that the flavour structure of κ_{ijkl} and κ'_{ijkl} is the same as in the minimal SU(5) model. This is an obvious consequence of the way the SM matter fields are embedded into SO(10) representations.

the above operators involve a kaon, and in practice $p \to K^+ \bar{\nu}$ dominates. The corresponding amplitude is obtained by "dressing" the D=5 operators of Eq. (B.5) with gaugino/higgsino loops. If the first two generation squarks are close in mass, as the observed weakness of hadronic flavour-violating processes may suggest, the dominant contributions come from the wino dressing of the $QQQL$ operator and from the charged higgsino dressing of the $u^c u^c d^c e^c$ operator. Here we consider only the former contribution; the latter (which is significant for the $K^+ \bar{\nu}_\tau$ channel only) would give a stronger constraint on M_T only for large values of $\tan\beta$. The proton partial decay width then reads:

$$\begin{aligned}\Gamma(p \to K^+ \bar{\nu}) &= \frac{(m_p^2 - m_{K^+}^2)^2}{32\pi f_\pi^2 m_p^3}\, |\beta_H|^2 \\ &\times \sum_l \left| \left[1 + \frac{m_p}{3 m_\Lambda}\,(D+3F) \right] C_{112l} + \left[\frac{m_p}{2 m_{\Sigma^0}}\,(D-F) + \frac{m_p}{6 m_\Lambda}\,(D+3F) \right] C_{121l} \right|^2 , \quad \text{(B.7)}\end{aligned}$$

where $f_\pi = 130$ MeV is the pion decay constant; β_H is the hadronic parameter defined by $\beta_H P_L u_p = \langle 0 | (u_L d_L) u_L | p \rangle$, where u_p is the proton spinor and the parenthesis indicates the contraction of Lorentz indices; D and F are chiral Lagrangian parameters; and C_{ijkl} is the Wilson coefficient of the four-fermion operator $(u_{Li} d_{Lj})(d_{Lk} \nu_l)$. The most recent lattice determination of β_H is $|\beta_H| = 0.0120(26)\,\mathrm{GeV}^3$ [48], while an analysis of hyperon decay measurements gives $F + D = 1.2670 \pm 0.0030$ and $F - D = -0.341 \pm 0.016$ [49]. The Wilson coefficients C_{ijkl} read:

$$C_{ijkl} = \frac{\alpha_2}{4\pi}\, A_R \sum_{m,n} V_{mj} V_{nk} (\kappa_{imnl} - \kappa_{nmil}) \left[f(\tilde{d}'_{Li}, \tilde{u}_{Lm}) + f(\tilde{u}_{Ln}, \tilde{e}_{Ll}) \right] , \quad \text{(B.8)}$$

where A_R is a renormalization factor, V is the CKM matrix, the κ coefficients are expressed in the basis $q = (u, d') \equiv (u, Vd)$, and the loop function f is given by (M_2 is the wino mass):

$$f(a,b) = \frac{M_2}{m_a^2 - m_b^2} \left[\frac{m_a^2}{m_a^2 - M_2^2} \ln\left(\frac{m_a^2}{M_2^2}\right) - \frac{m_b^2}{m_b^2 - M_2^2} \ln\left(\frac{m_b^2}{M_2^2}\right) \right] . \quad \text{(B.9)}$$

For degenerate sfermion masses ($m_a = m_b \equiv \tilde{m}$) and a hierarchy $M_2^2 \ll \tilde{m}^2$, $f(a,b)$ reduces to $M_2/\tilde{m}^2$ to a good approximation. In Eq. (B.8), the quantity $(\kappa_{imnl} - \kappa_{nmil})$ is evaluated at the GUT scale, where the operator $QQQL$ is generated. The renormalization factor $A_R = A_{SD} A_{LD}$ contains a short-distance piece A_{SD} which accounts for the renormalization of the superpotential operator $QQQL$ from M_{GUT} to the supersymmetry breaking scale (here identified with m_Z), and a long-distance piece A_{LD} which encodes the renormalization of the four-fermion operator $(u_{Li} d_{Lj})(d_{Lk} \nu_l)$ from m_Z to $\mu_{had} = 1$ GeV. The latter is given by:

$$A_{LD} = \left(\frac{\alpha_3(\mu_{had})}{\alpha_3(m_c)} \right)^{2/9} \left(\frac{\alpha_3(m_c)}{\alpha_3(m_b)} \right)^{6/25} \left(\frac{\alpha_3(m_b)}{\alpha_3(m_Z)} \right)^{6/23} \simeq 1.4 , \quad \text{(B.10)}$$

and the former by (neglecting the Yukawa contributions):

$$A_{SD} = \prod_I \left(\frac{\alpha_1(M_I)}{\alpha_1(M_{I+1})} \right)^{-1/5 b_1^I} \left(\frac{\alpha_2(M_I)}{\alpha_2(M_{I+1})} \right)^{-3/b_2^I} \left(\frac{\alpha_3(M_I)}{\alpha_3(M_{I+1})} \right)^{-4/b_3^I} , \quad \text{(B.11)}$$

where I runs over all mass thresholds M_I between m_Z and M_{GUT}, and $b^I_{1,2,3}$ are the beta function coefficients between M_I and M_{I+1}. Given the hierarchy among the quark Yukawa

couplings at the GUT scale, one has $C_{112l} \simeq C_{121l}$, with:

$$C_{112l} \simeq \frac{\alpha_2}{4\pi} A_R \left(M_T^{-1}\right)_{T^{10}\overline{T}^{16}} \frac{\lambda_{d_l} V_{ul}^* \, e^{-i\Phi_l^d}}{\sin\theta_H} \sum_n e^{2i\Phi_n^u} \lambda_{u_n} V_{nd} V_{ns} \left[f(\tilde{d}'_L, \tilde{u}_{Ln}) + f(\tilde{u}_{Ln}, \tilde{e}_{Ll}) \right], \quad \text{(B.12)}$$

where Φ_n^u and Φ_n^d ($n = 1, 2, 3$, $\Phi_1^u + \Phi_2^u + \Phi_3^u = 0$) are high-energy phases, and the sum is dominated by the $n = 2$ term (i.e. by the $\tilde{c}_L$ loops).

We are now in a position to derive a lower bound on $\left(M_T^{-1}\right)_{T^{10}\overline{T}^{16}}$ from the experimental constraint $\tau(p \to K^+\bar{\nu}) > 2.3 \times 10^{33}$ yrs (90% C.L.) [50]. For definiteness, we consider the model (ii) of Section 4.3 with $M_\Delta = 10^{12}$ GeV, $\tan\beta = 10$ and a spectrum close to the one of Fig. 6(b), with squark masses in the TeV range (except for the lightest stop), slepton masses around 750 GeV and $M_2 \approx 280$ GeV. This spectrum gives $f(\tilde{d}'_L, \tilde{c}_L) + f(\tilde{c}_L, \tilde{e}_{Ll}) \approx 0.45\,\text{TeV}^{-1}$. Evaluating the Yukawa couplings at the GUT scale, we find $\lambda_u V_{ud} V_{us} = 5.2 \times 10^{-7}$, $\lambda_c V_{cd} V_{cs} = -2.6 \times 10^{-4}$, $\lambda_t |V_{td} V_{ts}| = 7.5 \times 10^{-5}$, and $(\lambda_d V_{ud}, \lambda_s V_{us}, \lambda_b |V_{ub}|) = (0.43, 1.9, 2.0) \times 10^{-4}$. Depending on the values of the high-energy phases, a partial cancellation between the $\tilde{t}_L$ and the $\tilde{c}_L$ contributions is possible. Here we do not consider this possibility and keep only the dominant $\tilde{c}_L$ contribution, which gives:

$$\sqrt{\sum_l |C_{112l}|^2} \simeq 9.9 \times 10^{-13}\,\text{GeV}^{-1} \left(\frac{A_{SD}}{8.1}\right) \left(\frac{1}{\sin\theta_H}\right) \left(\frac{2\bar{f}}{0.45\,\text{TeV}^{-1}}\right) \left(M_T^{-1}\right)_{T^{10}\overline{T}^{16}}, \quad \text{(B.13)}$$

where we have replaced $f(\tilde{d}'_L, \tilde{c}_L) + f(\tilde{c}_L, \tilde{e}_{Ll})$ by an average value $2\bar{f}$, and $A_{SD} \simeq 8.1$ (to be compared with 7.8 in the MSSM with $\tan\beta = 10$) takes into account the various thresholds shown in Fig. 1(a). From Eqs. (B.7) and (B.13), we finally obtain:

$$\frac{\tau(p \to K^+\bar{\nu})}{2.3 \times 10^{33}\,\text{yrs}} = \left(\frac{0.012\,\text{GeV}^3}{|\beta_H|}\right)^2 \left(\frac{\sin\theta_H}{1}\right)^2 \left(\frac{0.45\,\text{TeV}^{-1}}{2\bar{f}}\right)^2 \left(\frac{(4.4 \times 10^{18}\,\text{GeV})^{-1}}{\left(M_T^{-1}\right)_{T^{10}\overline{T}^{16}}}\right)^2, \quad \text{(B.14)}$$

which yields the upper bound $\left(M_T^{-1}\right)_{T^{10}\overline{T}^{16}} \lesssim \left[\, 4.4 \times 10^{18}\,\text{GeV}\,(1/\sin\theta_H)\,(\tan\beta/10)\,\right]^{-1}$, where we have restored the approximate $\tan\beta$ dependence of the proton decay amplitude (one can indeed see from Eq. (B.12) that C_{112l} and C_{121l} scale as $1/\sin 2\beta$). This bound is a conservative one, and several effects could relax it: there are still large uncertainties on the hadronic parameter β_H; A_{SD} overestimates the running of the κ coefficients, since it does not include the Yukawa contribution; a superpartner spectrum characterized by heavier sfermions and/or a stronger sfermion/gaugino mass hierarchy would reduce the proton decay amplitude; corrections to the mass relation $M_d = M_e^T$ could affect the couplings of the Higgs triplets to quarks and leptons; and finally, some sfermion mixing patterns compatible with the observed level of flavour violation can significantly reduce the proton decay amplitude [51].

Let us now translate this bound into constraints on the doublet-triplet splitting parameters. From Eq. (B.2) we have:

$$\left(M_T^{-1}\right)_{T^{10}\overline{T}^{16}} = \frac{\sqrt{3}\,\overline{V}_1 \left(\sqrt{6}\,\bar{\eta} M_{10'} - \bar{\rho} h V_{B-L}\right)}{\sqrt{2}\, M_{16}\, (h V_{B-L})^2} = \frac{3 \tan\theta_H M_{10'}}{(h V_{B-L})^2} - \frac{\sqrt{3}\,\bar{\rho}\overline{V}_1}{\sqrt{2}\, M_{16}\, h V_{B-L}}, \quad \text{(B.15)}$$

where we made use of $V_R = M_{16}/2g$ and $\tan\theta_H = \bar{\eta}\overline{V}_1/M_{16}$. Since the SO(10) gauge symmetry is broken in one step (see Appendix A), $\overline{V}_1 \sim V_{B-L} \sim M_{16} \sim M_{\text{GUT}}$ and the desired suppression of $\left(M_T^{-1}\right)_{T^{10}\overline{T}^{16}}$ with respect to its natural value M_{GUT}^{-1} can be achieved by a partial cancellation

in the combination $(\sqrt{6}\,\bar{\eta} M_{10'} - \bar{\rho} h V_{B-L})$, or by taking $\bar{\rho} \ll 1$ and $M_{10'}$ in the $(10^{13} - 10^{14})$ GeV range, where the lower value corresponds to the conservative bound on $\left(M_T^{-1}\right)_{T^{10}\overline{T}^{16}}$. Note that $\tan\theta_H \ll 1$ would not help, since the upper bound on $\left(M_T^{-1}\right)_{T^{10}\overline{T}^{16}}$ becomes stricter for smaller values of θ_H. In this paper we take[12] $M_{10'} = 10^{14}$ GeV in order to avoid a strong cancellation among unrelated superpotential parameters. One can check that all colour Higgs triplets acquire GUT-scale masses in this case, while one pair of Higgs doublet sits at the intermediate scale $M_{10'}$. The consequences for gauge coupling unification are discussed in Appendix C, and summarized in Section 4.3.

B.3 Gauge-mediated proton decay

Let us now discuss the contribution of SO(10) gauge interactions to proton decay [47]. Due to the way the SM matter fields are embedded into SO(10) representations, there are no new gauge contributions with respect to the SU(5) case, i.e. only the X and Y gauge bosons mediate proton decay. The dominant decay mode from (X, Y) exchange is $p \to \pi^0 e^+$, and its rate is given by (neglecting the non-gauge contributions, which are subdominant):

$$\Gamma(p \to \pi^0 e^+) \;=\; \frac{(m_p^2 - m_{\pi^0}^2)^2}{64\pi f_\pi^2 m_p^3}\, |\alpha_H|^2 \,(1 + D + F)^2 \left(|C_{RL}|^2 + |C_{LR}|^2\right), \tag{B.16}$$

where α_H is the hadronic parameter defined by $\alpha_H P_L u_p = \langle 0|(u_R d_R) u_L|p\rangle$, D and F are the same chiral Lagrangian parameters as above, and C_{RL} and C_{LR} are the Wilson coefficients of the four-fermion operators $(u_R d_R)(u_L e_L)$ and $(u_L d_L)(u_R e_R)$, respectively. The most recent lattice determination of α_H is $|\alpha_H| = 0.0112(25)\,\mathrm{GeV}^3$ [48]. The Wilson coefficients take the same form as in minimal SU(5):

$$C_{RL} \;=\; A_R\, \frac{g_{\rm GUT}^2}{M_V^2}\,, \qquad C_{LR} \;=\; A_R\, \frac{g_{\rm GUT}^2}{M_V^2} \left(1 + |V_{ud}|^2\right), \tag{B.17}$$

where $M_V \equiv M_X = M_Y$ is the mass of the heavy (X, Y) gauge bosons. The long-distance piece of the renormalization factor $A_R = A_{SD} A_{LD}$ is given by Eq. (B.10), while its short-distance piece reads:

$$A_{SD} \;=\; \prod_I \left(\frac{\alpha_1(M_I)}{\alpha_1(M_{I+1})}\right)^{-\frac{23}{30 b_1^I}} \left(\frac{\alpha_2(M_I)}{\alpha_2(M_{I+1})}\right)^{-\frac{3}{2 b_2^I}} \left(\frac{\alpha_3(M_I)}{\alpha_3(M_{I+1})}\right)^{-\frac{4}{3 b_3^I}}, \tag{B.18}$$

where, as in Eq. (B.11), the Yukawa contributions have been neglected, and I runs over all mass thresholds M_I between m_Z and $M_{\rm GUT}$. Together with Eq. (B.17), Eq. (B.16) gives:

$$\tau(p \to \pi^0 e^+) \;=\; \left(8.2 \times 10^{34}\,\mathrm{yrs}\right) \left(\frac{0.0112\,\mathrm{GeV}^3}{|\alpha_H|}\right)^2 \left(\frac{2.4}{A_{SD}}\right)^2 \left(\frac{1/24}{\alpha_{\rm GUT}}\right)^2 \left(\frac{M_V}{10^{16}\,\mathrm{GeV}}\right)^4, \tag{B.19}$$

to be compared with the experimental upper bound $\tau(p \to \pi^0 e^+) > 8.2 \times 10^{33}$ yrs (90% C.L.) [52]. In Eq. (B.19), $A_{SD} = 2.4$ and $\alpha_{\rm GUT} = 1/24$ are the reference MSSM values. Consider now the model (i) of Section 4.3 with $M_\Delta = M_\Sigma = 10^{12}$ GeV, $M_T = 10^{13}$ GeV, $M_H = 10^{14}$ GeV

[12] Note that the mass scale $M_{10'} \sim 10^{14}$ GeV can be generated by an interaction term $\lambda\, \mathbf{S10'10'}$ with $\lambda \sim 10^{-2}$, where $\mathbf{S}$ is the SO(10) singlet introduced in Appendix A.

and the same heavy fermion spectrum as in Fig. 1(a). At the one-loop level and omitting both low-energy and GUT thresholds, one obtains $\alpha_{\rm GUT} = 1/14$, $M_{\rm GUT} = 3.5 \times 10^{15}$ GeV and $A_{SD} = 2.4$. These values are at odds with the experimental limit on the proton lifetime (assuming $M_V = M_{\rm GUT}$ leads to $\tau(p \to \pi^0 e^+) = 4.2 \times 10^{32}$ yrs). On the contrary, for model (ii) with the spectrum of Fig. 1(a) (with $M_\Delta = 10^{12}$ GeV, $M_\Sigma = M_S = M_T = M_O = 10^{13}$ GeV and $M_H = 10^{14}$ GeV), one finds $\alpha_{\rm GUT} = 1/12$, $M_{\rm GUT} = 1.2 \times 10^{16}$ GeV and $A_{SD} = 2.5$, leading to $\tau(p \to \pi^0 e^+) = 3.9 \times 10^{34}$ yrs for $M_V = M_{\rm GUT}$. We shall therefore adopt model (ii) as our reference model, although model (i) may be viable if 2-loop running and threshold effects conspire to increase the GUT scale. Also, the corrections needed to depart from the minimal SU(5) relation $M_d = M_e^T$ will in general modify Eq. (B.17) by introducing non-trivial fermion mixing angles in the heavy gauge boson couplings, and this could significantly reduce the proton decay rate [53].

B.4 Proton decay from non-renormalizable operators

For completeness, we mention that proton decay could also be induced by non-renormalizable superpotential operators of the form:

$$\frac{d_{ijkl}}{\Lambda^2}\, \mathbf{16}_i \mathbf{16}_j \mathbf{16}_k \mathbf{10}_l \overline{\mathbf{16}}\,, \tag{B.20}$$

where Λ is the cutoff, and the SU(5)-singlet component of the $\overline{\mathbf{16}}$ acquires a GUT-scale vev. These operators, if present, will generate the $D = 5$ operators of Eq. (B.5) after SO(10) symmetry breaking. In order to avoid a confict with the experimental bound on proton lifetime, they must either be forbidden by some symmetry, or the ones involving light generation fields must be suppressed by small coefficients d_{ijkl}. This is actually what one would expect in a theory of flavour capable of explaining the hierarchy of fermion masses. Note that the required suppression of the coefficients is less severe than in conventional SO(10) models, where the dangerous operators, of the form $\mathbf{16}_i \mathbf{16}_j \mathbf{16}_k \mathbf{16}_l$, have dimension 5.

C Intermediate scales and gauge coupling unification

In this appendix, we study the constraints imposed by the requirement of successful gauge coupling unification on the extra heavy states present below the GUT scale, using 1-loop renormalization group equations.

As shown in Appendix B.2, the experimental constraint on the $p \to K^+ \bar\nu$ rate can easily be satisfied by allowing a pair of Higgs doublets to lie at an intermediate scale. This tends to spoil unification, but we will show that the problem can be cured by splitting the components of the **54**. In fact, it is also desirable to give GUT-scale masses to some components of the SU(5) multiplets $(\mathbf{15}, \overline{\mathbf{15}})$ and **24** contained in the **54** in order to preserve perturbative unification. Indeed, the presence of additional chiral superfields at intermediate scales increases the value of the unified gauge coupling $\alpha_{\rm GUT}$ with respect to the MSSM. One has to check that a Landau pole is not reached soon above (or even below) the GUT scale, because this would make the predictions of the model very sensitive to the effects of higher-dimensional operators. In the SO(10) scenario studied in this paper, keeping the full **54** close to the seesaw scale would give $\alpha_{\rm GUT} \simeq 1/4$, and the Landau pole would be reached at a scale $\Lambda \approx 2 M_{\rm GUT}$. Thus, motivated both by perturbativity and by proton decay, we are led to split the $(\mathbf{15}, \overline{\mathbf{15}})$ and **24** multiplets, keeping below the GUT scale only the components that are necessary for the

seesaw mechanism and for leptogenesis, together with possible additional components needed to achieve unification.

As is customary, we define $M_{\rm GUT}$ as the scale where α_1 and α_2 meet. Denoting by M_n the masses of the intermediate-scale states and by $b_i^{(n)}$ $(i = 1,2,3)$ their contributions to the beta-function coefficients, the 1-loop predictions for $\alpha_3(m_Z)$, $M_{\rm GUT}$ and $\alpha_{\rm GUT}$ read:

$$\frac{1}{\alpha_3(m_Z)} - \frac{1}{\alpha_3^0(m_Z)} = \frac{1}{2\pi}\sum_n \frac{b_{32}^{(n)} b_{21} - b_{21}^{(n)} b_{32}}{b_{21}} \ln \frac{M^0_{\rm GUT}}{M_n} \ , \tag{C.1}$$

$$\ln \frac{M_{\rm GUT}}{M^0_{\rm GUT}} = -\sum_n \frac{b_{21}^{(n)}}{b_{21}} \ln \frac{M^0_{\rm GUT}}{M_n} \ , \tag{C.2}$$

$$\frac{1}{\alpha_{\rm GUT}} - \frac{1}{\alpha^0_{\rm GUT}} = \frac{1}{2\pi}\sum_n \frac{b_2 b_{21}^{(n)} - b_2^{(n)} b_{21}}{b_{21}} \ln \frac{M^0_{\rm GUT}}{M_n} \ , \tag{C.3}$$

where the superscript "0" refers to MSSM quantities, with $(b^0_1, b^0_2, b^0_3) = (\frac{33}{5}, 1, -3)$, $M^0_{\rm GUT} \simeq 2 \times 10^{16}$ GeV, $\alpha^0_{\rm GUT} \simeq 1/24$, and

$$b_{ij} \equiv b_i - b_j \ , \qquad b_i \equiv b^0_i + \sum_n b_i^{(n)} \ . \tag{C.4}$$

Given the fact that the MSSM prediction for $\alpha_3(m_Z)$ significantly deviates from the measured value (using 2-loop RGEs and including low-energy supersymmetric thresholds, one finds $[1/\alpha_3^{\rm exp}(m_Z) - 1/\alpha_3^0(m_Z)] \simeq +4.3/(2\pi)$, which corresponds to a 5σ deviation), a positive contribution of the extra states to Eq. (C.1) would be welcome.

Replacing Eq. (C.2) into Eq. (C.3) one finds:

$$\frac{1}{\alpha_{\rm GUT}} = \frac{1}{\alpha^0_{\rm GUT}} - \frac{b_2}{2\pi} \ln \frac{M_{\rm GUT}}{M^0_{\rm GUT}} - \sum_n \frac{b_2^{(n)}}{2\pi} \ln \frac{M^0_{\rm GUT}}{M_n} \ . \tag{C.5}$$

The third term always increases $\alpha_{\rm GUT}$, while the second one can decrease or increase it, depending on whether $M_{\rm GUT}$ is smaller or larger than $M^0_{\rm GUT}$. However, the contribution of the second term is bounded by the requirement $M_{\rm GUT} \gtrsim 5 \times 10^{15}$ GeV coming from proton decay (see Apprendix B.3). Therefore, in order to avoid a too large value of $\alpha_{\rm GUT}$, the additional intermediate-scale fields besides the ones needed to realize leptogenesis and to suppress proton decay [namely $(\Delta, \overline{\Delta})$, S and/or T, $(\mathbf{5}_i, \overline{\mathbf{5}}_i)$ and $(H, \overline{H})$] should better be $SU(2)_L$ singlets. There are two such **54** components: $(\Sigma, \overline{\Sigma})$ and O. Since O is an electroweak singlet, it does not affect $M_{\rm GUT}$ nor $\alpha_{\rm GUT}$ (but it corrects the prediction for α_3). Adding only O would push $M_{\rm GUT}$ above the Planck scale. We are thus left with two possibilities: (i) intermediate $(\Sigma, \overline{\Sigma})$; (ii) intermediate $(\Sigma, \overline{\Sigma})$ and O. The right-hand side of Eq. (C.1) reads, for each of the two cases:

$$\text{(i)} \quad \frac{1}{10\pi}\left[41 \ln \frac{M^0_{\rm GUT}}{M_\Sigma} - 22 \ln \frac{M^0_{\rm GUT}}{M_\Delta} - 20 \ln \frac{M^0_{\rm GUT}}{M_T} - 7 \ln \frac{M^0_{\rm GUT}}{M_H}\right] , \tag{C.6a}$$

$$\text{(ii)} \quad \frac{1}{10\pi}\left[33 \ln \frac{M^0_{\rm GUT}}{M_\Sigma} - 21 \ln \frac{M^0_{\rm GUT}}{M_\Delta} - 15 \ln \frac{M^0_{\rm GUT}}{M_T} - 6 \ln \frac{M^0_{\rm GUT}}{M_H} + 15 \ln \frac{M^0_{\rm GUT}}{M_O}\right] , \tag{C.6b}$$

while $M_{\rm GUT}$ and $\alpha_{\rm GUT}$ are given by the same expression in both cases:

$$\ln\frac{M_{\rm GUT}}{M^0_{\rm GUT}} = \frac{1}{15}\left[-8\ln\frac{M^0_{\rm GUT}}{M_\Sigma}+\ln\frac{M^0_{\rm GUT}}{M_\Delta}+5\ln\frac{M^0_{\rm GUT}}{M_T}+\ln\frac{M^0_{\rm GUT}}{M_H}\right], \tag{C.7}$$

$$\frac{1}{\alpha_{\rm GUT}}-\frac{1}{\alpha^0_{\rm GUT}} = \frac{1}{30\pi}\left[88\ln\frac{M^0_{\rm GUT}}{M_\Sigma}-71\ln\frac{M^0_{\rm GUT}}{M_\Delta}-85\ln\frac{M^0_{\rm GUT}}{M_T}-26\ln\frac{M^0_{\rm GUT}}{M_H}\right.$$
$$\left.-15\ln\frac{(M^0_{\rm GUT})^3}{M_{5_1}M_{5_2}M_{5_3}}\right], \tag{C.8}$$

where we assumed $M_{L_i}=M_{D^c_i}\equiv M_{5_i}$. From the above equations, we can see that lowering M_Σ decreases $\alpha_{\rm GUT}$ as desired (and improves the prediction for α_3), but decreases $M_{\rm GUT}$.

Let us first consider case (i). Assuming $M_\Sigma=M_\Delta\equiv M_{15}=10^{12}$ GeV, $M_T\equiv M_{24}=10^{13}$ GeV and $M_H=10^{14}$ GeV (as well as $\lambda_H=0.045$, $V_1=M_{\rm GUT}$, $\tan\beta=10$ and $\tan\theta_H=1$ to fix the masses of the heavy $(\mathbf{5}_i,\bar{\mathbf{5}}_i)$ pairs), we obtain, at the 1-loop level:

$$\frac{1}{\alpha_3(m_Z)}-\frac{1}{\alpha^0_3(m_Z)} = -\frac{0.19}{2\pi}, \quad M_{\rm GUT} = 3.5\times10^{15}\ \text{GeV}, \quad \frac{1}{\alpha_{\rm GUT}} = 14.1\,. \tag{C.9}$$

In this case, the prediction for $\alpha_3(m_Z)$ is only slightly larger than in the MSSM, and the value of the unified coupling remains reasonable. Unfortunately, the unification scale lies almost one order of magnitude below the MSSM prediction, which leads to a too fast proton decay rate (see Appendix B.3). Gauge coupling unification is approximately preserved if M_{15} and M_{24} are varied while keeping the ratio M_{24}/M_{15} fixed. In particular, increasing M_{15} while keeping $M_{24}/M_{15}=10$ slightly increases $M_{\rm GUT}$ (and decreases α_3 and $\alpha_{\rm GUT}$). For instance, for $M_{15}=10^{13}$ GeV, one obtains $1/\alpha_3(m_Z)-1/\alpha^0_3(m_Z)=+0.27/(2\pi)$, $M_{\rm GUT}=4.8\times10^{15}$ GeV and $\alpha_{\rm GUT}=1/15.9$. Still the unification scale is dangerously close to the proton decay bound.

Let us now consider case (ii). As anticipated, only $\alpha_3(m_Z)$ is affected by the presence of O below $M_{\rm GUT}$. This gives the possibility of increasing $M_{\rm GUT}$ with respect to case (i) by increasing M_Σ, while correcting the prediction for $\alpha_3(m_Z)$ by adjusting M_O. For instance, the choice $M_\Delta=10^{12}$ GeV, $M_T=M_O=M_\Sigma=10^{13}$ GeV and $M_H=10^{14}$ GeV gives, at the 1-loop level:

$$\frac{1}{\alpha_3(m_Z)}-\frac{1}{\alpha^0_3(m_Z)} = +\frac{2.2}{2\pi}, \quad M_{\rm GUT} = 1.2\times10^{16}\ \text{GeV}, \quad \frac{1}{\alpha_{\rm GUT}} = 12.5\,. \tag{C.10}$$

In this case, unification works better than in the MSSM. Indeed, we checked that the prediction for $\alpha_3(m_Z)$, including low-energy thresholds and the 2-loop MSSM running, matches the measured value within 1σ. Moreover, there is no conflict between the value of $M_{\rm GUT}$ and proton decay, and the Landau pole lies one order of magnitude above $M_{\rm GUT}$. From Eq. (C.6b), we can see that the contributions of T and O cancel if $M_T=M_O$. Therefore, unification is still preserved if the masses of the various states are varied while keeping $M_T=M_O$ and $M^3_\Sigma/M^2_\Delta\approx10^{15}$ GeV.

The splitting of the masses of the $\mathbf{54}$ components needed to realize case (ii) can be achieved with the set of operators shown in Table 3, where V_{PS} is the Pati-Salam invariant vev of the $\mathbf{54}'$, V_{B-L} is the vev of the $\mathbf{45}_2$, which is aligned along the $B-L$ direction, and $\mathbf{45}_3$ has no vev (see Appendix A). At the renormalizable level, only $(Z,\bar Z)$ and $(V,\bar V)$ acquire a (GUT-scale) mass. The dimension-5 operators provide $M_S=M_T=M_O\sim M^2_{\rm GUT}/\Lambda$ as well as $M_\Sigma\sim M^2_{\rm GUT}/\Lambda$ (more precisely, when the GUT-scale masses of the $\mathbf{45}_3$ and $\mathbf{54}'$ components are taken into account, $M_{S,T,O}$ and M_Σ are further suppressed by a mild seesaw-like mechanism). Finally, the dimension-6 operator generates $M_\Delta\sim M^3_{\rm GUT}/\Lambda^2$.

Operator	Massive **54** components	Mass
$\mathbf{54}.\mathbf{45}_3.\mathbf{54}'$	$(Z,\overline{Z}),(V,\overline{V})$	V_{PS}
$\frac{1}{\Lambda}(\mathbf{54}.\mathbf{45}_3)_{\mathbf{45}}(\mathbf{16}.\overline{\mathbf{16}})_{\mathbf{45}}$	S,T,O	$\frac{1}{\Lambda}V_1\overline{V}_1$
$\frac{1}{\Lambda}(\mathbf{54}.\mathbf{45}_2)_{\mathbf{54}}(\mathbf{54}'.\mathbf{54}')_{\mathbf{54}}$	$(\Sigma,\overline{\Sigma})$	$\frac{1}{\Lambda}V_{B-L}V_{PS}$
$\frac{1}{\Lambda^2}(\mathbf{54}.\mathbf{54})_{\mathbf{1}}(\overline{\mathbf{16}}.\mathbf{45}_2.\mathbf{16})_{\mathbf{1}}$	$(\Delta,\overline{\Delta})$	$\frac{1}{\Lambda^2}V_1\overline{V}_1V_{B-L}$

Table 3: The operators listed in the first column generate masses for the **54** components given in the second column. The order of magnitude of these masses is reported in the third column.

D Renormalization group equations

Below $M_{\rm GUT}$, the superpotential terms (2) and (3) read, in terms of the SM components (4):

$$\begin{aligned}
W &= (\lambda_u)_{ij}\, u^c_i q_j h_u + (\lambda_d)_{ij}\, d^c_i q_j h_d + (\lambda_e)_{ij}\, e^c_i l_j h_d \\
&+ (\hat\lambda_d)_{ij}\, D^c_i q_j h_d + (\hat\lambda_e)_{ij}\, e^c_i L_j h_d + (M_L)_{ij} L_i \bar L_j + (M_{D^c})_{ij} D^c_i \bar D^c_j \\
&+ \frac{1}{2}\,(f_\Delta)_{ij}\, l_i \Delta l_j + \frac{1}{2}\,(f_{\bar\Delta})_{ij}\, \bar L_i \bar\Delta \bar L_j + \frac{1}{\sqrt2}\,(f_Z)_{ij}\, d^c_i Z l_j + \frac{1}{\sqrt2}\,(f_{\bar Z})_{ij}\, \bar D^c_i \bar Z \bar L_j \\
&+ \frac{1}{2}\,(f_\Sigma)_{ij}\, d^c_i \Sigma d^c_j + \frac{1}{2}\,(f_{\bar\Sigma})_{ij}\, \bar D^c_i \bar\Sigma \bar D^c_j + \frac{1}{\sqrt2}\,(f_V)_{ij}\, d^c_i V \bar L_j + \frac{1}{\sqrt2}\,(f_{\bar V})_{ij}\, \bar D^c_i \bar V l_j \\
&+ \frac{1}{\sqrt2}\,(f_O)_{ij}\, d^c_i O \bar D^c_j + \frac{1}{\sqrt2}\,(f_T)_{ij}\, \bar L_i T l_j + \sqrt{\frac{3}{20}}\,(f_{S_l})_{ij}\, \bar L_i S l_j - \frac{1}{\sqrt{15}}\,(f_{S_d})_{ij}\, d^c_i S \bar D^c_j \\
&+ \frac{1}{\sqrt2}\,\sigma_T\, h_u T h_d + \sqrt{\frac{3}{20}}\,\sigma_S\, h_u S h_d + \frac{1}{2}\,\sigma_\Delta\, h_d \Delta h_d + \frac{1}{2}\,\sigma_{\bar\Delta}\, h_u \bar\Delta h_u \\
&+ M_\Delta \Delta\bar\Delta + M_Z Z\bar Z + M_\Sigma \Sigma\bar\Sigma + M_V V\bar V + \frac{1}{2}(M_S\, S^2 + M_T\, T^2 + M_O\, O^2)\ ,
\end{aligned} \tag{D.1}$$

where Clebsch-Gordan coefficients have been factorized out, and contractions of $SU(3)_C$ and $SU(2)_L$ indices are understood. The interactions of fields with GUT-scale masses, namely the right-handed neutrinos and the components of **16** and **10** other than the light Higgs doublets, have been omitted[13]. The boundary conditions at the GUT scale for the superpotential couplings and mass parameters are:

$$\begin{aligned}
&\lambda_u = y\ ,\quad \lambda_e = \lambda_d^T = \sin\theta_H h\ ,\quad \hat\lambda_e = \hat\lambda_d = \cos\theta_H y\ , \\
&f_X = f \quad \text{for } X = \Delta,\bar\Delta,Z,\bar Z,\Sigma,\bar\Sigma,V,\bar V,O,T,S_l,S_d\ , \\
&\sigma_\Delta = \cos^2\theta_H\sigma\ ,\quad \sigma_{\bar\Delta} = \sigma\ ,\quad \sigma_T = \sigma_S = \cos\theta_H\sigma\ , \\
&M_L = M_{D^c} = hV_1\ ,
\end{aligned} \tag{D.2}$$

where the Higgs mixing angle θ_H is defined by Eq. (B.3), and V_1 is the vev of the $SU(5)$ singlet component of the **16**. We did not write boundary conditions for the masses of the **54**

[13] Similarly, the interactions of the **54** components with GUT-scale masses should not appear in Eq. (D.1), nor in the RGEs.

components, since, as discussed in Appendix C, they are assumed to be split by operators not included in the superpotential (3).

The soft supersymmetry breaking terms for the SO(10) fields are defined by:

$$
\begin{aligned}
-L_{soft} \;=\; & (m^2_{16})_{ij}\mathbf{16}^*_i\mathbf{16}_j + (m^2_{10})_{ij}\mathbf{10}^*_i\mathbf{10}_i + m^2_{16_H}\mathbf{16}^*\mathbf{16} + m^2_{10_H}\mathbf{10}^*\mathbf{10} + m^2_{54}\mathbf{54}^*\mathbf{54} \\
& + \left(\frac{1}{2}(A_y)_{ij}\mathbf{16}_i\mathbf{16}_j\mathbf{10} + (A_h)_{ij}\mathbf{16}_i\mathbf{10}_j\mathbf{16} + \frac{1}{2}(A_f)_{ij}\mathbf{10}_i\mathbf{10}_j\mathbf{54} + \frac{1}{2}A_\sigma\mathbf{10}\,\mathbf{10}\,\mathbf{54} + \text{h.c.}\right) \\
& + \left(\frac{1}{2}M_{1/2}\lambda^a\lambda^a + \text{h.c.}\right) , \qquad \text{(D.3)}
\end{aligned}
$$

where we used the same notation for the chiral multiplets and for their scalar components, λ^a are the SO(10) gauginos, and we omitted the B-terms. The soft terms of the SM components are given, at the GUT scale, by the following boundary conditions:

$$
\begin{aligned}
& m^2_q = m^{2T}_{u^c} = m^{2T}_{e^c} = m^2_L = m^{2T}_{D^c} = m^2_{16} \,, \\
& m^2_l = m^{2T}_{d^c} = m^2_{\bar{L}} = m^{2T}_{\bar{D}^c} = m^2_{10} \,, \\
& m^2_{h_u} = m^2_{10_H} \,, \quad m^2_{h_d} = \cos^2\theta_H\, m^2_{10_H} + \sin^2\theta_H\, m^2_{16_H} \,, \\
& m^2_X = m^2_{54} \quad \text{for } X = \Delta, \bar{\Delta}, Z, \bar{Z}, \Sigma, \bar{\Sigma}, S, T, O, V, \bar{V} \,, \\
& M_1 = M_2 = M_3 = M_{1/2} \,.
\end{aligned} \qquad \text{(D.4)}
$$

The boundary conditions for the A-terms, not included in the above list, are analogous to the ones for the corresponding Yukawa couplings. One has for instance:

$$
A_u = A_y \;, \quad A_e = A^T_d = \sin\theta_H A_h \;, \quad \hat{A}_e = \hat{A}_d = \cos\theta_H A_y \;. \qquad \text{(D.5)}
$$

In the numerical study of Section 5, we solved the 1-loop RGEs for all superpotential couplings and soft terms below M_{GUT}. For brevity, we only list below the RGEs for the MSSM parameters, which are sufficient for a leading-log analysis of flavour-violating effects. Let us first recall the 1-loop RGEs for gauge couplings and gaugino masses:

$$
\frac{d}{dt}\,\alpha_a^{-1} \;=\; -\frac{1}{2\pi}b_a \;, \qquad b_a \;=\; -3\,C_2(G_a) + \sum_R T_a(R) \;, \qquad \text{(D.6)}
$$

$$
\frac{d}{dt}M_a \;=\; \frac{1}{2\pi}b_a\alpha_a M_a \;, \qquad \text{(D.7)}
$$

where $t = \log(\mu/\mu_0)$, μ being the renormalization scale and μ_0 a reference scale, $C_2(G_a)$ is the second Casimir invariant of the group G_a, $T_a(R)$ is the Dynkin index of the representation R, and the sum in b_a runs over all chiral superfields with mass smaller than μ.

The 1-loop RGEs for the MSSM Yukawa couplings are:

$$
\begin{aligned}
(4\pi)^2\frac{d}{dt}\lambda_u \;=\; & \lambda_u\left(3\lambda_u^\dagger\lambda_u + \lambda_d^\dagger\lambda_d + \hat{\lambda}_d^\dagger\hat{\lambda}_d\right) + \text{Tr}\left(3\lambda_u\lambda_u^\dagger\right)\lambda_u \\
& + \left(\frac{3}{4}\left|\sigma_T\right|^2 + \frac{3}{20}\left|\sigma_S\right|^2 + \frac{3}{2}\left|\sigma_{\bar{\Delta}}\right|^2\right)\lambda_u - \left(\frac{13}{15}g_1^2 + 3g_2^2 + \frac{16}{3}g_3^2\right)\lambda_u \;, \qquad \text{(D.8)}
\end{aligned}
$$

$$(4\pi)^2\frac{d}{dt}\lambda_d = \lambda_d\left(3\lambda_d^\dagger\lambda_d+\lambda_u^\dagger\lambda_u+3\hat\lambda_d^\dagger\hat\lambda_d\right)+\mathrm{Tr}\left(3\lambda_d\lambda_d^\dagger+\lambda_e\lambda_e^\dagger+\hat\lambda_e\hat\lambda_e^\dagger+3\hat\lambda_d\hat\lambda_d^\dagger\right)\lambda_d$$
$$+\left(f_Zf_Z^\dagger+f_Vf_V^\dagger+2f_\Sigma f_\Sigma^\dagger+\frac{4}{3}f_Of_O^\dagger+\frac{1}{15}f_{S_d}f_{S_d}^\dagger\right)\lambda_d$$
$$+\left(\frac{3}{4}|\sigma_T|^2+\frac{3}{20}|\sigma_S|^2+\frac{3}{2}|\sigma_\Delta|^2\right)\lambda_d-\left(\frac{7}{15}g_1^2+3g_2^2+\frac{16}{3}g_3^2\right)\lambda_d\ , \qquad \text{(D.9)}$$

$$(4\pi)^2\frac{d}{dt}\lambda_e = \left(3\lambda_e\lambda_e^\dagger+3\hat\lambda_e\hat\lambda_e^\dagger\right)\lambda_e+\mathrm{Tr}\left(3\lambda_d\lambda_d^\dagger+\lambda_e\lambda_e^\dagger+\hat\lambda_e\hat\lambda_e^\dagger+3\hat\lambda_d\hat\lambda_d^\dagger\right)\lambda_e$$
$$+\lambda_e\left(\frac{3}{2}f_\Delta^\dagger f_\Delta+\frac{3}{2}f_Z^\dagger f_Z+\frac{3}{2}f_{\bar V}^\dagger f_{\bar V}+\frac{3}{4}f_T^\dagger f_T+\frac{3}{20}f_{S_l}^\dagger f_{S_l}\right)$$
$$+\left(\frac{3}{4}|\sigma_T|^2+\frac{3}{20}|\sigma_S|^2+\frac{3}{2}|\sigma_\Delta|^2\right)\lambda_e-\left(\frac{9}{5}g_1^2+3g_2^2\right)\lambda_e\ . \qquad \text{(D.10)}$$

The 1-loop RGEs for the MSSM soft sfermion masses are:

$$(4\pi)^2\frac{d}{dt}m_q^2 = \left(m_q^2\lambda_u^\dagger\lambda_u+\lambda_u^\dagger\lambda_um_q^2+2\lambda_u^\dagger\lambda_um_{h_u}^2+2\lambda_u^\dagger m_{u^c}^2\lambda_u+2A_u^\dagger A_u\right)$$
$$+\left(m_q^2\lambda_d^\dagger\lambda_d+\lambda_d^\dagger\lambda_dm_q^2+2\lambda_d^\dagger\lambda_dm_{h_d}^2+2\lambda_d^\dagger m_{d^c}^2\lambda_d+2A_d^\dagger A_d\right)$$
$$+\left(m_q^2\hat\lambda_d^\dagger\hat\lambda_d+\hat\lambda_d^\dagger\hat\lambda_dm_q^2+2\hat\lambda_d^\dagger\hat\lambda_dm_{h_d}^2+2\hat\lambda_d^\dagger m_{D^c}^2\hat\lambda_d+2\hat A_d^\dagger\hat A_d\right)$$
$$-\left(\frac{2}{15}|M_1|^2g_1^2+6|M_2|^2g_2^2+\frac{32}{3}|M_3|^2g_3^2-\frac{1}{5}g_1^2S\right)\ , \qquad \text{(D.11)}$$

$$(4\pi)^2\frac{d}{dt}m_{d^c}^2 = 2\left(m_{d^c}^2\lambda_d\lambda_d^\dagger+\lambda_d\lambda_d^\dagger m_{d^c}^2+2\lambda_d\lambda_d^\dagger m_{h_d}^2+2\lambda_dm_q^2\lambda_d^\dagger+2A_dA_d^\dagger\right)$$
$$+\left(m_{d^c}^2f_Zf_Z^\dagger+f_Zf_Z^\dagger m_{d^c}^2+2f_Zf_Z^\dagger m_Z^2+2f_Zm_l^2f_Z^\dagger+2A_{f_Z}A_{f_Z}^\dagger\right)$$
$$+2\left(m_{d^c}^2f_\Sigma f_\Sigma^\dagger+f_\Sigma f_\Sigma^\dagger m_{d^c}^2+2f_\Sigma f_\Sigma^\dagger m_\Sigma^2+2f_\Sigma m_{d^c}^{2T}f_\Sigma^\dagger+2A_{f_\Sigma}A_{f_\Sigma}^\dagger\right)$$
$$+\frac{1}{15}\left(m_{d^c}^2f_{S_d}f_{S_d}^\dagger+f_{S_d}f_{S_d}^\dagger m_{d^c}^2+2f_{S_d}f_{S_d}^\dagger m_S^2+2f_{S_d}m_{\bar D^c}^{2T}f_{S_d}^\dagger+2A_{f_{S_d}}A_{f_{S_d}}^\dagger\right)$$
$$+\frac{4}{3}\left(m_{d^c}^2f_Of_O^\dagger+f_Of_O^\dagger m_{d^c}^2+2f_Of_O^\dagger m_O^2+2f_Om_{\bar D^c}^{2T}f_O^\dagger+2A_{f_O}A_{f_O}^\dagger\right)$$
$$+\left(m_{d^c}^2f_Vf_V^\dagger+f_Vf_V^\dagger m_{d^c}^2+2f_Vf_V^\dagger m_V^2+2f_Vm_{\bar L}^2f_V^\dagger+2A_{f_V}A_{f_V}^\dagger\right)$$
$$+2\left(m_{d^cD^c}^2\hat\lambda_d\lambda_d^\dagger+\lambda_d\hat\lambda_d^\dagger m_{d^cD^c}^{2\dagger}\right)-\left(\frac{8}{15}|M_1|^2g_1^2+\frac{32}{3}|M_3|^2g_3^2-\frac{2}{5}g_1^2S\right)\ , \qquad \text{(D.12)}$$

$$(4\pi)^2\frac{d}{dt}m_{u^c}^2 = 2\left(m_{u^c}^2\lambda_u\lambda_u^\dagger+\lambda_u\lambda_u^\dagger m_{u^c}^2+2\lambda_u\lambda_u^\dagger m_{h_u}^2+2\lambda_um_q^2\lambda_u^\dagger+2A_uA_u^\dagger\right)$$
$$-\left(\frac{32}{15}|M_1|^2g_1^2+\frac{32}{3}|M_3|^2g_3^2+\frac{4}{5}g_1^2S\right)\ , \qquad \text{(D.13)}$$

$$
\begin{aligned}
(4\pi)^2\frac{d}{dt}m_l^2 \;=\; & \left(m_l^2\lambda_e^\dagger\lambda_e + \lambda_e^\dagger\lambda_e m_l^2 + 2\lambda_e^\dagger\lambda_e m_{h_d}^2 + 2\lambda_e^\dagger m_{e^c}^2\lambda_e + 2A_e^\dagger A_e\right) \\
& + \frac{3}{2}\left(m_l^2 f_\Delta^\dagger f_\Delta + f_\Delta^\dagger f_\Delta m_l^2 + 2f_\Delta^\dagger f_\Delta m_\Delta^2 + 2f_\Delta^\dagger m_l^{2T} f_\Delta + 2A_{f_\Delta}^\dagger A_{f_\Delta}\right) \\
& + \frac{3}{2}\left(m_l^2 f_Z^\dagger f_Z + f_Z^\dagger f_Z m_l^2 + 2f_Z^\dagger f_Z m_Z^2 + 2f_Z^\dagger m_{d^c}^2 f_Z + 2A_{f_Z}^\dagger A_{f_Z}\right) \\
& + \frac{3}{2}\left(m_l^2 f_{\bar V}^\dagger f_{\bar V} + f_{\bar V}^\dagger f_{\bar V} m_l^2 + 2f_{\bar V}^\dagger f_{\bar V} m_{\bar V}^2 + 2f_{\bar V}^\dagger m_{\bar D^c}^2 f_{\bar V} + 2A_{f_{\bar V}}^\dagger A_{f_{\bar V}}\right) \\
& + \frac{3}{4}\left(m_l^2 f_T^\dagger f_T + f_T^\dagger f_T m_l^2 + 2f_T^\dagger f_T m_T^2 + 2f_T^\dagger m_{\bar L}^{2T} f_T + 2A_{f_T}^\dagger A_{f_T}\right) \\
& + \frac{3}{20}\left(m_l^2 f_{S_l}^\dagger f_{S_l} + f_{S_l}^\dagger f_{S_l} m_l^2 + 2f_{S_l}^\dagger f_{S_l} m_S^2 + 2f_{S_l}^\dagger m_{\bar L}^{2T} f_{S_L} + 2A_{f_{S_l}}^\dagger A_{f_{S_l}}\right) \\
& + \left(\lambda_e^\dagger\hat\lambda_e m_{lL}^{2\dagger} + m_{lL}^2\hat\lambda_e^\dagger\lambda_e\right) - \left(\frac{6}{5}|M_1|^2 g_1^2 + 6|M_2|^2 g_2^2 + \frac{3}{5}g_1^2 S\right)\ , \qquad \text{(D.14)}
\end{aligned}
$$

$$
\begin{aligned}
(4\pi)^2\frac{d}{dt}m_{e^c}^2 \;=\; & 2\left(m_{e^c}^2\lambda_e\lambda_e^\dagger + \lambda_e\lambda_e^\dagger m_{e^c}^2 + 2\lambda_e\lambda_e^\dagger m_{h_d}^2 + 2\lambda_e m_l^2\lambda_e^\dagger + 2A_e A_e^\dagger\right) \\
& + 2\left(m_{e^c}^2\hat\lambda_e\hat\lambda_e^\dagger + \hat\lambda_e\hat\lambda_e^\dagger m_{e^c}^2 + 2\hat\lambda_e\hat\lambda_e^\dagger m_{h_d}^2 + 2\hat\lambda_e m_L^2\hat\lambda_e^\dagger + 2\hat A_e\hat A_e^\dagger\right)_{ij} \\
& - \left(\frac{24}{5}|M_1|^2 g_1^2 - \frac{6}{5}g_1^2 S\right)\ , \qquad \text{(D.15)}
\end{aligned}
$$

where the hypercharge D-term contribution S is given by:

$$
\begin{aligned}
S \;=\; & m_{h_u}^2 - m_{h_d}^2 + \mathrm{Tr}(-m_l^2 + m_{e^c}^2 - 2m_{u^c}^2 + m_{d^c}^2 + m_q^2) + \mathrm{Tr}(-m_L^2 + m_{D^c}^2 + m_{\bar L}^2 - m_{\bar D^c}^2) \\
& + 3(m_\Delta^2 - m_{\bar\Delta}^2) + (m_Z^2 - m_{\bar Z}^2) + 4(m_{\bar\Sigma}^2 - m_\Sigma^2) + 5(m_V^2 - m_{\bar V}^2)\ . \qquad \text{(D.16)}
\end{aligned}
$$

The mixing terms $m_{d^cD^c}^2 d^{c*}D^c + m_{lL}^2 l^*L + \text{h.c.}$ appearing in the RGEs for $m_{d^c}^2$ and m_l^2, although absent at $M_{\rm GUT}$, are generated by the RGE evolution and must be included for consistency.

Finally, the 1-loop RGEs for the MSSM A-terms are:

$$
\begin{aligned}
(4\pi)^2\frac{d}{dt}A_u \;=\; & A_u\left(5\lambda_u^\dagger\lambda_u + \lambda_d^\dagger\lambda_d + \hat\lambda_d^\dagger\hat\lambda_d\right) + 4\lambda_u\lambda_u^\dagger A_u + 2\lambda_u\left(\lambda_d^\dagger A_d + \hat\lambda_d^\dagger\hat A_d\right) \\
& + \mathrm{Tr}\left(3\lambda_u\lambda_u^\dagger\right)A_u + \left(\frac{3}{4}|\sigma_T|^2 + \frac{3}{20}|\sigma_S|^2 + \frac{3}{2}|\sigma_{\bar\Delta}|^2\right)A_u \\
& + \mathrm{Tr}\left(6A_u\lambda_u^\dagger\right)\lambda_u + \left(\frac{3}{2}A_{\sigma_T}\sigma_T^* + \frac{3}{10}A_{\sigma_S}\sigma_S^* + 3A_{\sigma_{\bar\Delta}}\sigma_{\bar\Delta}^*\right)\lambda_u \\
& + \frac{13}{15}g_1^2\left(2M_1\lambda_u - A_u\right) + 3g_2^2\left(2M_2\lambda_u - A_u\right) + \frac{16}{3}g_3^2\left(2M_3\lambda_u - A_u\right)\ , \qquad \text{(D.17)}
\end{aligned}
$$

$$
\begin{aligned}
(4\pi)^2\frac{d}{dt}A_d \;=\;& A_d\left(5\lambda_d^\dagger\lambda_d+\lambda_u^\dagger\lambda_u+5\hat{\lambda}_d^\dagger\hat{\lambda}_d\right)+4\lambda_d\lambda_d^\dagger A_d+2\lambda_d\left(\lambda_u^\dagger A_u+2\hat{\lambda}_d^\dagger\hat{A}_d\right)\\
&+\left(f_Zf_Z^\dagger+f_Vf_V^\dagger+2f_\Sigma f_\Sigma^\dagger+\frac{4}{3}f_Of_O^\dagger+\frac{1}{15}f_{S_d}f_{S_d}^\dagger\right)A_d\\
&+\left(2A_{f_Z}f_Z^\dagger+2A_{f_V}f_V^\dagger+4A_{f_\Sigma}f_\Sigma^\dagger+\frac{8}{3}A_{f_O}f_O^\dagger+\frac{2}{15}A_{f_{S_d}}f_{S_d}^\dagger\right)\lambda_d\\
&+\mathrm{Tr}\left(3\lambda_d\lambda_d^\dagger+\lambda_e\lambda_e^\dagger+\hat{\lambda}_e\hat{\lambda}_e^\dagger+3\hat{\lambda}_d\hat{\lambda}_d^\dagger\right)A_d+\left(\frac{3}{4}\left|\sigma_T\right|^2+\frac{3}{20}\left|\sigma_S\right|^2+\frac{3}{2}\left|\sigma_\Delta\right|^2\right)A_d\\
&+2\,\mathrm{Tr}\left(3A_d\lambda_d^\dagger+A_e\lambda_e^\dagger+\hat{A}_e\hat{\lambda}_e^\dagger+3\hat{A}_d\hat{\lambda}_d^\dagger\right)\lambda_d+\left(\frac{3}{2}A_{\sigma_T}\sigma_T^*+\frac{3}{10}A_{\sigma_S}\sigma_S^*+3A_{\sigma_\Delta}\sigma_\Delta^*\right)\lambda_d\\
&+\frac{7}{15}g_1^2\left(2M_1\lambda_d-A_d\right)+3g_2^2\left(2M_2\lambda_d-A_d\right)+\frac{16}{3}g_3^2\left(2M_3\lambda_d-A_d\right)\;, \qquad \text{(D.18)}
\end{aligned}
$$

$$
\begin{aligned}
(4\pi)^2\frac{d}{dt}A_e \;=\;& 4\left(\lambda_e\lambda_e^\dagger+\hat{\lambda}_e\hat{\lambda}_e^\dagger\right)A_e+5A_e\lambda_e^\dagger\lambda_e+5\hat{A}_e\hat{\lambda}_e^\dagger\lambda_e\\
&+A_e\left(\frac{3}{2}f_\Delta^\dagger f_\Delta+\frac{3}{2}f_Z^\dagger f_Z+\frac{3}{2}f_{\bar{V}}^\dagger f_{\bar{V}}+\frac{3}{4}f_T^\dagger f_T+\frac{3}{20}f_{S_l}^\dagger f_{S_l}\right)\\
&+\lambda_e\left(3f_\Delta^\dagger A_{f_\Delta}+3f_Z^\dagger A_{f_Z}+3f_{\bar{V}}^\dagger A_{f_{\bar{V}}}+\frac{3}{2}f_T^\dagger A_{f_T}+\frac{3}{10}f_{S_l}^\dagger A_{f_{S_l}}\right)\\
&+\mathrm{Tr}\left(3\lambda_d\lambda_d^\dagger+\lambda_e\lambda_e^\dagger+\hat{\lambda}_e\hat{\lambda}_e^\dagger+3\hat{\lambda}_d\hat{\lambda}_d^\dagger\right)A_e+\left(\frac{3}{4}\left|\sigma_T\right|^2+\frac{3}{20}\left|\sigma_S\right|^2+\frac{3}{2}\left|\sigma_\Delta\right|^2\right)A_e\\
&+2\,\mathrm{Tr}\left(3A_d\lambda_d^\dagger+A_e\lambda_e^\dagger+\hat{A}_e\hat{\lambda}_e^\dagger+3\hat{A}_d\hat{\lambda}_d^\dagger\right)\lambda_e+\left(\frac{3}{2}A_{\sigma_T}\sigma_T^*+\frac{3}{10}A_{\sigma_S}\sigma_S^*+3A_{\sigma_\Delta}\sigma_\Delta^*\right)\lambda_e\\
&+\frac{9}{5}g_1^2\left(2M_1\lambda_e-A_e\right)+3g_2^2\left(2M_2\lambda_e-A_e\right)\;. \qquad \text{(D.19)}
\end{aligned}
$$

References

[1] S. Weinberg, Phys. Rev. Lett. **43** (1979) 1566.

[2] P. Minkowski, Phys. Lett. B **67** (1977) 421; M. Gell-Mann, P. Ramond and R. Slansky, in *Supergravity*, P. van Nieuwenhuizen and D.Z. Freedman (eds.), North Holland Publ. Co., 1979, p. 315; T. Yanagida, in *Proc. of the Workshop on the Baryon Number of the Universe and Unified Theories*, O. Sawada and A. Sugamoto (eds.), Tsukuba, Japan, 13-14 Feb. 1979, p. 95; S. L. Glashow, in *Quarks and Leptons*, Cargèse Lectures, 9-29 July 1979, Plenum, New York, 1980, p. 687; R. N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. **44** (1980) 912.

[3] M. Fukugita and T. Yanagida, Phys. Lett. B **174** (1986) 45.

[4] F. Borzumati and A. Masiero, Phys. Rev. Lett. **57** (1986) 961.

[5] M. Magg and C. Wetterich, Phys. Lett. B **94** (1980) 61; G. Lazarides, Q. Shafi and C. Wetterich, Nucl. Phys. B **181** (1981) 287; R. N. Mohapatra and G. Senjanovic, Phys. Rev. D **23** (1981) 165. See also J. Schechter and J. W. F. Valle, Phys. Rev. D **22** (1980) 2227.

[6] H. Georgi, AIP Conf. Proc. **23** (1975) 575; H. Fritzsch and P. Minkowski, Annals Phys. **93** (1975) 193.

[7] M. Frigerio, P. Hosteins, S. Lavignac and A. Romanino, Nucl. Phys. B **806** (2009) 84 [arXiv:0804.0801 [hep-ph]].

[8] P. J. O'Donnell and U. Sarkar, Phys. Rev. D **49** (1994) 2118 [arXiv:hep-ph/9307279]; E. Ma and U. Sarkar, Phys. Rev. Lett. **80** (1998) 5716 [arXiv:hep-ph/9802445]; T. Hambye and G. Senjanovic, Phys. Lett. B **582** (2004) 73 [arXiv:hep-ph/0307237].

[9] A. Rossi, Phys. Rev. D **66** (2002) 075003 [arXiv:hep-ph/0207006].

[10] J. Hisano, H. Murayama and T. Yanagida, Phys. Rev. D **49** (1994) 4966; Z. Berezhiani and Z. Tavartkiladze, Phys. Lett. B **409** (1997) 220 [arXiv:hep-ph/9612232]; Y. Nomura and T. Yanagida, Phys. Rev. D **59** (1999) 017303 [arXiv:hep-ph/9807325]; J. L. Rosner, Phys. Rev. D **61** (2000) 097303; Z. Berezhiani and A. Rossi, Nucl. Phys. B **594** (2001) 113 [arXiv:hep-ph/0003084]; T. Asaka, Phys. Lett. B **562** (2003) 291 [arXiv:hep-ph/0304124]; K. S. Babu, I. Gogoladze, P. Nath and R. M. Syed, Phys. Rev. D **74** (2006) 075004 [arXiv:hep-ph/0607244]; S. M. Barr, Phys. Rev. D **76** (2007) 105024 [arXiv:0706.1490 [hep-ph]]; M. Malinsky, Phys. Rev. D **77** (2008) 055016.

[11] J. A. Casas and A. Ibarra, Nucl. Phys. B **618** (2001) 171 [arXiv:hep-ph/0103065].

[12] F. R. Joaquim and A. Rossi, Phys. Rev. Lett. **97** (2006) 181801 [arXiv:hep-ph/0604083], Nucl. Phys. B **765** (2007) 71 [arXiv:hep-ph/0607298].

[13] R. Barbieri and L. J. Hall, Phys. Lett. B **338** (1994) 212 [arXiv:hep-ph/9408406]; R. Barbieri, L. J. Hall and A. Strumia, Nucl. Phys. B **445** (1995) 219 [arXiv:hep-ph/9501334].

[14] L. J. Hall, V. A. Kostelecky and S. Raby, Nucl. Phys. B **267** (1986) 415.

[15] T. Schwetz, M. A. Tortola and J. W. F. Valle, New J. Phys. **10** (2008) 113011 [arXiv:0808.2016 [hep-ph]].

[16] I. Masina and C. A. Savoy, Nucl. Phys. B **661**, 365 (2003) [arXiv:hep-ph/0211283].

[17] P. Paradisi, JHEP **0510**, 006 (2005) [arXiv:hep-ph/0505046].

[18] M. Ciuchini, A. Masiero, P. Paradisi, L. Silvestrini, S. K. Vempati and O. Vives, Nucl. Phys. B **783**, 112 (2007) [arXiv:hep-ph/0702144].

[19] F. Gabbiani, E. Gabrielli, A. Masiero and L. Silvestrini, Nucl. Phys. B **477** (1996) 321 [arXiv:hep-ph/9604387].

[20] M. Bona *et al.*, arXiv:0709.0451 [hep-ex].

[21] P. Brax and C. A. Savoy, Nucl. Phys. B **447** (1995) 227 [arXiv:hep-ph/9503306].

[22] I. Masina, Nucl. Phys. B **671** (2003) 432 [arXiv:hep-ph/0304299]; Y. Farzan and M. E. Peskin, Phys. Rev. D **70** (2004) 095001 [arXiv:hep-ph/0405214].

[23] J. Hisano, M. Nagai and P. Paradisi, arXiv:0812.4283 [hep-ph].

[24] M. Pospelov and A. Ritz, Annals Phys. **318** (2005) 119 [arXiv:hep-ph/0504231].

[25] B. C. Regan, E. D. Commins, C. J. Schmidt and D. DeMille, Phys. Rev. Lett. **88** (2002) 071805.

[26] G. W. Bennett *et al.* [Muon (g-2) Collaboration], arXiv:0811.1207 [hep-ex].

[27] C. A. Baker *et al.*, Phys. Rev. Lett. **97** (2006) 131801 [arXiv:hep-ex/0602020].

[28] A. J. Buras and D. Guadagnoli, Phys. Rev. D **78** (2008) 033005 [arXiv:0805.3887 [hep-ph]]; A. J. Buras and D. Guadagnoli, arXiv:0901.2056 [hep-ph].

[29] S. Dimopoulos and F. Wilczek, preprint NSF-ITP-82-07, August 1981 (unpublished) and *Supersymmetric Unified Models*, in *Proceedings of the Erice Summer School* (A. Zichichi ed.), 1981, pp. 237-249; M. Srednicki, Nucl. Phys. B **202** (1982) 327; K. S. Babu and S. M. Barr, Phys. Rev. D **48** (1993) 5354 [arXiv:hep-ph/9306242].

[30] M. L. Brooks *et al.* [MEGA Collaboration], Phys. Rev. Lett. **83** (1999) 1521 [arXiv:hep-ex/9905013].

[31] U. Bellgardt *et al.* [SINDRUM Collaboration], Nucl. Phys. B **299** (1988) 1.

[32] C. Dohmen *et al.* [SINDRUM II Collaboration.], Phys. Lett. B **317**, 631 (1993); J. Kaulard *et al.* [SINDRUM II Collaboration], Phys. Lett. B **422**, 334 (1998).

[33] B. Aubert [The BABAR Collaboration], arXiv:0908.2381 [hep-ex].

[34] C. Amsler *et al.* [Particle Data Group], Phys. Lett. B **667** (2008) 1.

[35] E. Barberio *et al.* [Heavy Flavor Averaging Group (HFAG) Collaboration], arXiv:0704.3575 [hep-ex].

[36] T. Aaltonen *et al.* [CDF Collaboration], Phys. Rev. Lett. **100** (2008) 101802 [arXiv:0712.1708 [hep-ex]].

[37] J. Hisano, T. Moroi, K. Tobe and M. Yamaguchi, Phys. Rev. D **53**, 2442 (1996) [arXiv:hep-ph/9510309].

[38] G. Isidori and A. Retico, JHEP **0209** (2002) 063 [arXiv:hep-ph/0208159]; G. Isidori and P. Paradisi, Phys. Lett. B **639** (2006) 499 [arXiv:hep-ph/0605012].

[39] G. Degrassi, P. Gambino and P. Slavich, Comput. Phys. Commun. **179**, 759 (2008) [arXiv:0712.3265 [hep-ph]].

[40] M. Ciuchini *et al.*, JHEP **9810** (1998) 008 [arXiv:hep-ph/9808328].

[41] L. M. Barkov *et al.*, PSI Proposal R-99-05 (1999); S. Ritt [MEG Collaboration], Nucl. Phys. Proc. Suppl. **162**, 279 (2006); J. Adam *et al.* [MEG collaboration], arXiv:0908.2594 [hep-ex].

[42] R. M. Carey *et al.* [Mu2e Collaboration], "Proposal to search for $\mu^- N \to e^- N$ with a single event sensitivity below 10^{-16}".

[43] The PRIME working group, "Search for the $\mu - e$ Conversion Process at an Ultimate Sensitivity of the Order of 10^{18} with PRISM", unpublished; LOI to J-PARC 50-GeV PS, LOI-25, `http://psux1.kek.jp/~jhf-np/LOIlist/LOIlist.html`

[44] J. R. Ellis, D. V. Nanopoulos and S. Rudaz, Nucl. Phys. B **202** (1982) 43.

[45] P. Nath, A. H. Chamseddine and R. L. Arnowitt, Phys. Rev. D **32** (1985) 2348.

[46] J. Hisano, H. Murayama and T. Yanagida, Nucl. Phys. B **402** (1993) 46 [arXiv:hep-ph/9207279].

[47] For a recent review on proton decay, see P. Nath and P. Fileviez Perez, Phys. Rept. **441** (2007) 191 [arXiv:hep-ph/0601023].

[48] Y. Aoki *et al.* [RBC-UKQCD Collaboration], Phys. Rev. D **78** (2008) 054505 [arXiv:0806.1031 [hep-lat]].

[49] N. Cabibbo, E. C. Swallow and R. Winston, Ann. Rev. Nucl. Part. Sci. **53** (2003) 39 [arXiv:hep-ph/0307298].

[50] K. Kobayashi *et al.* [Super-Kamiokande Collaboration], Phys. Rev. D **72** (2005) 052007 [arXiv:hep-ex/0502026].

[51] B. Bajc, P. Fileviez Perez and G. Senjanovic, Phys. Rev. D **66** (2002) 075005 [arXiv:hep-ph/0204311].

[52] H. Nishino *et al.* [Super-Kamiokande Collaboration], Phys. Rev. Lett. **102** (2009) 141801 [arXiv:0903.0676 [hep-ex]].

[53] I. Dorsner and P. Fileviez Perez, Phys. Lett. B **625** (2005) 88 [arXiv:hep-ph/0410198].